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Volume XXVIII Number 7

Edited by William David Reeve

Third Report of the Committee on Geometry*

By RALPH BEATLEY
Harvard Graduate School of Education
Cambridge, Massachusetts

Report of the Committee of Ten (of the N.E.A.) on Secondary School Studies. American Book Company, 1894.

Recommends (p. 106) that "instruction in concrete geometry, with numerous exercises, be introduced into the grammar school to familiarize the pupil with the facts of plane and solid geometry." Recommends also that algebra "should be studied five hours a week during the first year,[1] and for about two hours and a half a week during the two years next succeeding; that the study of demonstrative geometry should begin at the end of the first year's study of algebra, and be carried on by the side of algebra for the next two years, occupying about two hours and a half a week."

Report of the Committee on College Entrance Requirements, in the Journal of Addresses and Proceedings of the N.E.A., 1899, p. 649.

Calls attention to three stages in instruction in geometry: concrete geometry, the introduction to demonstrative geometry, and

* The first part of this report appeared in the October Number of *The Mathematics Teacher.* Reprints of the entire report may be had postpaid by sending 35¢ to *The Mathematics Teacher,* 525 West 120th Street, New York, N. Y.

[1] From the context it is clear that this means the ninth grade. Later (on p. 111) the possibility of beginning algebra in grade 8 is recognized.

demonstrative geometry, thus anticipating stages A, B, C of the British Report on the Teaching of Geometry in Schools (1923).

Recommends parallel instruction in algebra and geometry throughout grades 7, 8, 9, 10. This accepts the recommendation of a collaborating committee of the American Mathematical Society, which defined its position (p. 775) as follows: "The two subjects (algebra and geometry) should be carried on simultaneously in each year of the remainder of the four years. By simultaneously is meant simply in the same year. It is not necessary that the hours of instruction be given to each alternately. The division may even be the first half-year to one and the second half-year to the other, but this arrangement is not to be preferred."

This committee of the American Mathematical Society says further (p. 773). "The attempt has been successfully made to teach geometry by interweaving solid and plane geometry from the outset. While the committee is not prepared to commend this, there are advantages to be gained by beginning solid geometry before plane geometry is completed. In the opinion of the committee, the restriction of the study in geometry in many secondary schools to plane geometry is unfortunate, and it is desirable that the school course and the college entrance requirement in geometry should cover both plane and solid geometry."

Final Report of the National Committee of Fifteen on Geometry Syllabus, 1912.

H. E. Slaught (Chairman), William Betz, Edward L. Brown, Charles L. Bouton, Florian Cajori, William Fuller, W. W. Hart, Herbert E. Hawkes, Earle R. Hedrick, Frederick E. Newton, Henry L. Rietz, Robert L. Short, David Eugene Smith, Eugene R. Smith, and Mabel Sykes.

The historical introduction of this report was prepared by Florian Cajori. He considers the change in attitude toward Euclid in France, Germany, Italy, England, and the United States. The committee stands for neither extreme formalism nor extreme utilitarianism, but rather for a reasonable attention to exercises with concrete setting and also a "quickening of the logical sense through a more rational distribution of emphasis which will make economy of both time and mental energy in mastering standard theorems and leave opportunity for a broader view of the subject in its concrete relations."

They submit a list of acceptable axioms and postulates, definitions, and types of propositions that are better passed over by the beginner without formal statement. They suggest that there is a list well upward of one hundred propositions which should continue to be proved (whether in the Euclid or Legendre arrangement).

With respect to incommensurables they wish to relieve the schools of the necessity of teaching the subject while leaving them free to do so if they wish. The committee suggests that preliminary work in geometry be done in the elementary grades; that the work in geometry be preceded by some definite work in geometric drawing; that geometry and algebra, or geometry and trigonometry be united. They recommend that plane geometry be assigned not less than one year nor more than one and one half years in the curriculum, preceded by algebra unless carried along with algebra. They also recommend a psychological presentation of exercises in which the more difficult ones are either postponed to a later part of the course or omitted altogether and the easier ones are brought into more immediate connection with the theorems to which they are related. They discuss not only the distribution of exercises, but also the grading and the nature of the same, and the sources of problems, also problems involving loci and algebraic methods. They conclude the report with lists of theorems, not exhaustive, which may be increased at the discretion of the teacher.

Report of the National Committee on Mathematical Requirements—The Reorganization of Mathematics in Secondary Education. 1923.

Outlines material in intuitive geometry, numerical trigonometry, and the barest elements of demonstrative geometry for the junior high school (pp. 22–25). Suggests the scope of plane demonstrative geometry in the senior high school (pp. 34, 35), and of solid geometry (p. 36). Includes syllabi in plane and solid geometry (pp. 55–63).

Recommends that "No attempt be made to define completely such terms as space, magnitude, distance, and solid, although the significance of such terms should be made clear by informal explanation and discussion." The terms circle and circumference should be considered as the *curve* and the *length of the curve*, respectively; similar considerations should govern the usage of the terms connected with the polygon. Such terms as isosceles triangle, rec-

tangle, and parallelogram should be made as general as possible. Recommends that the following terms be abandoned: antecedent, consequent, third and fourth proportional, equivalent, trapezium.

College Entrance Examination Board. Document 108—Definition of the Requirements in Geometry. 1923.

"Apart from the examination requirement, it is in the interest of sound mathematical training that each class of pupils receive the best and most thorough instruction in the theory of limits that they can accept. An immeasurably improved understanding of the meaning of limits has been perhaps the most important achievement of mathematical science in the past hundred years. As this improved knowledge becomes generally available, the importance of the subject for elementary instruction will increase rather than diminish."

"The Board wishes to accord all due latitude in the treatment of the subject of solid geometry. It recognizes the value of the further training in logical demonstration which supplements the study of plane geometry and is given in standard courses at the present time. It recognizes also that the intuitive geometry of the early school course may well be carried further as regards both a firmer grasp on space relations and the visualization of space figures, and the mensuration of surfaces and solids in space."

A Committee of the Mathematical Association (Great Britain)—The Teaching of Geometry in Schools. G. Bell and Sons, London, 1923.

Enunciates three stages in the teaching of geometry: Stage A, experimental, with emphasis on the unconscious acquisition of geometric knowledge; Stage B, deductive, but accepting obvious truths intuitively rather than proving them—as for example the theorems on vertical angles, on parallels, on congruence and similarity—and making occasional excursions into solid geometry; Stage C, the systematising stage, which emphasizes the sequence of theorems as part of a logical system and proves some of the propositions taken for granted intuitively in Stage B (pp. 14–22).

Recommends abolition of superposition at any stage and suggests the possibility of replacing the parallel postulate by a postulate of similarity. These ends can be achieved through assuming two general principles (p. 35):

1) Any figure (plane or solid) can be exactly reproduced anywhere. (Principle of Congruence.)

2) Any figure can be reproduced anywhere on any enlarged or diminished scale. (Principle of Similarity.)

"There is an irreconcilable divergence of aim between the pedagogue who wishes the basis of his teaching to be as broad as possible and the logician whose greatest triumph is to make one axiom serve where two have previously been used." (p. 40)

"The early work in trigonometry takes its place naturally in the course in elementary geometry, though the two subjects will subsequently become separate. The use of trigonometry in elementary geometry should not pass the level at which an *immediate* reinterpretation by means of similar right-angled triangles is possible; human nature takes refuge only too readily in a formula. To prohibit the use of trigonometry seems a misuse of authority, but in giving bookwork the teacher has the opportunity, by insisting that formulae and results are not everything, to supply a necessary counterpoise." (p. 60)

"There has been a tendency of recent years to reduce almost to zero the number of propositions (proved by the Indirect Method). But this must not be carried too far. The method of *reductio ad absurdum* is the recognized geometrical method for converse theorems. The method should be learnt, but the Committee hesitates to recommend any definite place for its first introduction." (p. 62)

II. Books on Method

Young, J. W. A. *The Teaching of Mathematics in the Elementary and the Secondary School.* Longmans, Green and Co., 1906, 1914, 1924.

There is so much material in a book of this type that it is not possible to treat its contents adequately and also briefly.

The author discusses the value of the study of mathematics, i.e. the utilitarian values, as a fundamental type of thought, as a tool for the study of nature, as exemplifying especially certain important modes of thought. The author distinguishes between the manner in which the subject matter is arranged and developed, which he calls method, and the manner in which it is presented to the pupils, which he calls mode. Then he discusses in greater detail the synthetic and analytic method, the deductive and the inductive

method, the socratic method, the heuristic method, and the laboratory method. As modes of instruction he considers the examination, the recitation, the lecture, the genetic, the individual, and the laboratory. His criterion for modes is: "if the mode used is such that the pupil makes no more progress than he would have made without the teacher, this on the face of the matter condemns that mode under the circumstances." Heading each chapter is an excellent bibliography.

In a discussion of the curriculum in mathematics, the author commends the simultaneous treatment of algebra and geometry. He thinks that the work would be enriched even more with a simultaneous treatment of mathematics and physics in the last if not the last two years of high school work.

"Definitions are an outgrowth of the work rather than the basis of it. There may be occasions when it would be better to accept a redundant definition than to allow the main line of progress to be diverted by consuming the time, energy, and attention necessary to trim and polish the redundant definition. Errors, or course, cannot be tolerated. In the case of teacher and pupil, the scope of the pupil's view should control the definition, not that of the teacher."

The author strongly favors the analytic method in geometry; also the teaching of informal geometry in the last year or two before the secondary school.

"It is also a good plan to omit temporarily in the first view of the subject those proofs which the teacher's experience has shown to be specially difficult for the pupil to grasp. With the clear statement that the proof is deferred, it is pedagogically sound to assume the truth of such propositions and use them freely. The strength gained from such use and from other work on the subject will enable the pupil later to master the omitted proof with ease."

"There is strong reason to expect that a moderate and judicious use of the simpler methods of modern geometry would render the work in geometry clearer, more systematic, and more attractive; and few would claim that we should not utilize the methods of modern geometry when they would prove serviceable. The application of symmetry may be taken as an example. On the other hand, it is very questionable whether place can profitably be found for specific topics of more recent geometry, as for example, the anharmonic ratio of four points on a straight line, or the modern geometry of the triangle."

"In the material world in which we live the Euclidean postulate seems to be satisfied; consequently Euclidean geometry will continue to be the geometry of practical life and hence of the schools, undisturbed by the discoveries resulting from the acute speculations of mathematicians." The use of motion, signed magnitudes, and algebraic methods is recommended in geometry proofs; and also work involving the space intuition of the pupil without taking up solid geometry proper.

Smith, David Eugene. *The Teaching of Geometry.* Ginn and Co., 1911.

This book is written for the large body of teachers who welcome the natural and gradual evolution of geometry toward better things . . . who are out of sympathy with the extreme of revolution or the extreme of stagnation.

"It stands for vitalizing geometry in every legitimate way; for improving the subject matter in such manner as not to destroy the pupil's interest; for so teaching geometry as to make it appeal to pupils as strongly as any other subject in the curriculum; but for the recognition of geometry for geometry's sake and not for the sake of fancied utility that hardly exists. Geometry is not studied, and never has been studied, because of its positive utility in commercial life or even in the workshop."

Preceding the detailed discussion of axioms, postulates, definitions, and leading theorems of seven books, there are chapters on certain questions now at issue, why geometry is studied, a brief history of geometry and of the teaching of geometry.

There is a chapter on efforts at improving Euclid in which the author discusses the attempts that have been made in England, the United States, France, and Germany. Of the efforts made in the United States, he discusses the Harvard Syllabus and that made by a committee of the Association of Mathematics Teachers in New England organized in 1904 as the most noteworthy.

"In preparing a syllabus no one should hope to bring the teaching world at once to agree to any great reduction in the number of basal propositions nor to agree to any radical change of terminology, symbolism, or sequence. Rather should it be the purpose to show that we have enough topics in geometry at present, and that the number of propositions is really greater than is absolutely necessary, so that teachers shall not be led to introduce any consider-

able number of propositions out of the large amount of new material that has recently been accumulating. Such a syllabus will always accomplish a good purpose, for at least it will provoke thought and arouse interest, but any other kind is bound to be ephemeral."

The majority of teachers lean toward the textbook in which the basal propositions are fully demonstrated as models and the original work for the student is secured through exercises. The author feels that books which suggest proofs are merely an attempt to take the place of the teacher and develop every lesson by the heuristic method; that they secure no more cooperative effort from the pupil than do the standard texts; and that textbooks of this sort tend to make it difficult for the student to read higher mathematics.

Schultze, Arthur. *The Teaching of Mathematics in Secondary Schools.* The Macmillan Co., 1914.

The author writes with the hope that "all important and fundamental topics have been treated in such detail as to be of real assistance to the inexperienced teacher." His object is to make the teaching of mathematics less informational and more disciplinary. He believes that the true value of mathematical study does not lie in its practical utility, and hence he cannot admit that the mensuration of parquet floors or the construction of window designs forms the true end of mathematical study. Among other things, the author includes a discussion of the methods of teaching mathematics, i.e. the synthetic and analytic methods, the inductive and deductive methods, the dogmatic and psychological methods, the lecture and heuristic methods and the laboratory method. He also discusses the foundations of mathematics and states that none of these facts relating to the foundations of mathematics has a place in the secondary schools because the student must be acquainted with a number of mathematical facts and theories before he can understand and appreciate investigations of so difficult a nature. Familiarity with the foundations is necessary to the teacher, however, showing him that perfect rigor exists only in the imagination.

There is detailed discussion of definitions in geometry, the first propositions, original exercises, equality of triangles, parallel lines, miscellaneous theorems, methods of attacking theorems and problems, limits, ratio and proportion, the regular polygons and

impossible constructions. Many exercises are given in detail and additional exercises are listed.

He considers translation, rotation about a point, and rotation about a line as the three methods of moving figures that are most useful. Under special devices he discusses similarity and multiplication of curves. He considers algebraic analysis a mode of attack that is not as interesting as the purely geometric ones, since it requires little originality and since the constructions thus obtained frequently lack elegance and clearness, but useful for the teacher who is obliged to get a solution of a problem in a short time.

Laisant, C.-A. *La Mathématique.* Gauthier-Villars, Paris, 1898, 1907.

Demands consideration of the geometry of one dimension before the geometry of two and three dimensions. Insists on necessity of taking straight line and plane as undefined terms. Notes that some postulates are but definitions in disguise. Welcomes departure from the rigid formalism of Legendre's geometry. Calls attention to the parallel between the geometry of the straight line and the algebra of positives and negatives; urges freer use of algebraic methods. Insists on the distinction between direct and symmetric equality in two dimensions, in three dimensions; likewise with respect to similitude. Parallel instruction in algebra and geometry makes it easy to introduce signed numbers and other algebraic methods into geometry. He would distinguish the area of triangle ABC from the area of triangle ACB, assigning sensed areas to these two configurations according to the sense of the angle at A. This would make area of ABC always equal to areas OAB+OBC+OCA regardless of the position of O. Recommends that trigonometry be regarded as a branch of geometry rather than as a separate subject. He would include homothety and inversion in the secondary course of study, and extols descriptive geometry as the best avenue to spatial visualization.

Young, J. W. *Fundamental Concepts of Algebra and Geometry.* Macmillan, 1911.

Sets forth the need of undefined terms, definitions, assumed propositions as necessary to any abstract logical system. A geometry is "true" only in relation to the undefined terms, etc., on which

it rests. Formal definitions should involve no idea more complex than the term to be defined. "No *formal* proof of any proposition should be attempted which seems obvious to the pupil without proof." (p. 56) To attempt to define everything formally, to prove everything formally, is "as absurd scientifically as it is pedagogically." It is pedagogically undesirable and scientifically unnecessary to reduce the number of undefined terms and unproved propositions to a minimum.

It is necessary, in the beginning, to make continued and insistent appeal to concrete geometric intuition. In a first course in geometry many assumptions essential to a purely formal logical development should be tacitly assumed.

Evans, G. W. *The Teaching of High School Mathematics.* Houghton Mifflin, 1911.

Emphasizes *number* in geometry as follows:

"Geometry is a method of investigating the shape and size of material things by means of diagrams, and of expressing the relations of shape and size among these diagrams by means of numbers."

"All the equations in geometry are really equations between numbers or combinations of numbers."

Treats incommensurables in terms of variable and limit defined in simple terms according to the spirit of modern analysis. Tantamount to defining irrational numbers in terms of rationals and assuming correspondence between real numbers and points on a line.

Recommends some trigonometry in geometry.

"A first treatment of geometry should not include proofs of theorems that do not cry for proofs."

"The execution of problems of construction in geometry has no logical connection with the development of theorems, except where the constructions show the existence of the figures referred to in the theorems. The question of the existence of such figures is settled by (exhibiting) extensive systems of theorems, and (hence) may safely be ignored. On the other hand, they do furnish excellent practice in the application of theorems, and should be freely used for this purpose."

Carson, G. St.L. *Mathematical Education.* Ginn, 1913.

"Children, when they commence mathematics, have formed

many intuitions concerning space and motion; are they to be adopted and used as postulates without question, to be tacitly ignored, or to be attacked?" To ask the pupil to demonstrate experimentally a familiar geometric fact by means of drawing and measurement is to attack his intuition. "A conscious induction from deliberate experiments is not an intuition." "Every intuition must be adopted as a postulate in the early treatment of any subject." "Deductions from the assumptions made should be rigorous; but in the earlier stages every acceptable statement or intuition should be taken as an assumption." "For example, in geometry the angle properties of parallel lines, properties of figures evident from symmetry, and the theory of similar figures (excluding areas) appear as postulates." (pp. 24–30) *The method of superposition* "should be once for all discarded as a proof." (p. 98) "His (Euclid's) motive enforces the introduction of solid geometry as late as possible, while ours demands it as early as possible." (p. 101) "It is advisable to ascertain whether, in the whole domain of geometrical knowledge, there may not be other matter of more educative value (than some of Euclid's geometry) within the grasp of ordinary boys and girls. My own conviction is that the elementary concepts and methods of projective geometry can be grasped by ordinary pupils, and that their educational value would be far greater." (p. 102)

Godfrey and Siddons. *The Teaching of Elementary Mathematics.* Cambridge University Press, 1931.

This book, for teachers, develops geometry for the boy from the age of 10 or 12 through the age of 15.

The work of the first year enlarges the geometric vocabulary, i.e., it enables the pupil to understand geometric words and use them correctly, though not necessarily to define them, presents scale drawing and the acquisition of geometric facts by discovery, encourages reasoning leading to discovery and the formulation of good English statements; logic is introduced in riders (not too formally). "Superposition is no proof."

The second year continues with parallelograms, midpoint theorem, ruler and compass constructions, loci, areas, Pythagorean Theorem, angle measurement and the like. (Stresses cyclic quadrilateral and angle in alternate segment.) There is also some three dimensional work involving the visualization of solid figures from two dimensional drawings and the actual drawing of diagrams of

solids. "It is unwise to make symmetry one of the fundamentals in a theoretical course" but "the boy's innate sense of symmetry should be developed" since it is "interesting and will ultimately give him a sense of power."

In the third year properties the areas of similar figures are developed and then at the age of 15 comes the age in which the boy systematizes his knowledge. Intellectual curiosity is now being stimulated in trigonometry, mechanics or calculus. Suggests that difficulties arise in theorems about the angles made by parallel lines and a transversal and about congruent triangles, but that these difficulties are philosophical and that the boy must be clear on what the theorems are; that they form part of the fundamental assumptions on which he will build; that gaps to be filled in center about inequality theorems, converse theorems, extension of Pythagorean Theorem and loci; that limits may be used to discover new theorems and to link various theorems together but should be avoided in formal proofs. The method for this stage is to look at a group of theorems and see how they hang together. A fixed sequence is not desirable. Trigonometry should be introduced into geometry.

Westaway, F. W. *Craftsmanship in the Teaching of Elementary Mathematics*. Blackie and Son, London, 1931.

"The difference between Euclid and the geometry now taught, which is still Euclidean geometry, is in the choice of working tools. In Euclid the proof of every proposition was ultimately traceable to the axioms; nowadays, the working tools consist of a small number of fundamental propositions. By means of carefully selected forms of practical work, the truth of these propositions is shown to beginners to be highly probable, but the formal proofs of such propositions are not considered until the boys reach the sixth form."

"Successful teachers of geometry seem to be those who have given special attention to the foundations of the subject, who possess exceptional ingenuity in making things clear, and who at an early stage make use of symmetry and of proportion and similarity. The teachers who are not successful are often those who confine their work to the limits of the ordinary textbook written for the use of boys; who fail to survey the whole geometrical field; who are still unacquainted with, or at all events, do not teach,

the great unifying principles of geometry, duality, continuity, symmetry, and so forth."

There follow fairly detailed accounts of early lessons in geometry. In discussing accuracy of drawing he says, "Exercises in accurate work of a more telling type may be found in the theorems of Brianchon, Desargues, Pascal, and others. As theorems, these are work for the upper forms; as geometrical constructions, they are useful in the lower middle forms, where they may be learnt as useful and interesting geometrical facts."

"A short course on simple projection at about the age of 13 helps later geometry enormously."

"Let logic of the strictly formal kind wait until foundations are well and truly laid."

In the discussion of lessons, solids are introduced early in preparatory geometry. There are also discussions of symmetry; congruent, symmetrical, and similar triangles; classifying and defining; parallel lines and transversals; proportion and similarity; golden section and the pentagon; the principle of continuity, which indicates a generalization of some fundamental principles; the principle of duality; solid geometry; elementary work in orthographic projection; first notions and main principles of radial projection.

"An elementary knowledge of the properties of lines and circles, of inversion, of conic sections, treated geometrically, of reciprocation and of harmonic section, ought to be expected from all second year sixth form boys."

Hassler and Smith. *The Teaching of Secondary Mathematics.* Macmillan, 1930.

"Transfer is more readily possible in the case of attitudes and ideals than in specific skills. If we wish transfer, however, we must teach with this idea definitely in mind. The problem of the teacher is to find the elements in his subject which are identical with elements in other fields and then stress these elements. Even then he must be careful to raise particular habits to ideals by taking suitable illustrations from other subjects and from the so-called daily lives of the pupils." (p. 123)

"We should teach demonstrative geometry in the senior high school mainly (though not exclusively) as a course in reasoning and aim to develop powers and habits of careful, accurate, and

independent thinking rather than to present the subject as a finished model of deductive logic." (p. 297)

"Begin demonstration by means of simple one-step and two-step exercises." (p. 328)

Cajori, F. *The Teaching and History of Mathematics in the United States*. Bureau of Education, Circular of Information No. 3, 1890.

(p. 129) "The following are the principal points of difference between Euclid and Legendre: (1) Legendre treats the theory of parallels differently; (2) Legendre does not give anything on proportion, but refers the student to algebra or arithmetic; (3) Euclid never supposes a line to be drawn until he has first demonstrated the possibility and shown the manner of drawing it. Legendre is not so scrupulous, but makes use of what are called 'hypothetical constructions'; (4) Legendre introduces new matter, especially in solid geometry, changes the order of propositions, and gives new definitions."

Stamper. *History of the Teaching of Elementary Geometry*. Teachers College, Columbia, 1909.

Legendre's "Éléments de géometrie" (1794) abandoned the sequence of Euclid, but was a logically sound presentation for the most part. He assumed the correspondence between line segment and number and based part of his geometry on arithmetic and algebra. He permitted "hypothetical constructions"; that is, he would admit the existence of the bisector of an angle in order to prove a certain theorem even though he had not yet demonstrated that the bisector could actually be constructed. He placed "geometry again on a sound logical basis, but differing from Euclid in order and method." (p. 81)

Lacroix commends the practice of indicating the analogies between plane and solid geometry, suggesting that "in some cases the proofs in the two geometries could be given in connection with each other."

Touton, F. C. *Solving Geometry Originals*. Bureau of Publications, Teachers College, Columbia University, 1924.

A set of geometry originals carefully prepared, given to students and the results tabulated and studied.

Those who make up examinations should write in detail each step of the solution requested and estimate the difficulty involved.

Boys seem to prefer construction exercises whereas girls more often select an exercise involving numerical solution or proof. On the score of abilities no evidence is found which demands segregation for the study of plane geometry. In the solution of construction exercises, girls have greater difficulty in distinguishing between given and required elements than do boys.

Teachers should recognize the individual differences of pupils. In lesson assignments the teacher must prepare each pupil for probable success in dealing with the exercises assigned for home-study by class instruction supplemented by necessary individual instruction.

"It is evident that pupils even after a year of study of geometry have not formed habits of examining their results to determine whether or not the evident and implied requirements of the exercise have been satisfied and the limiting conditions considered. Pupils should be taught to interpret and to check the results obtained from each solution. That such habits of dealing with results are not formed is obvious from this study."

Shibli, J. *Recent Developments in the Teaching of Geometry*. 1932.

This book considers geometry as a moving rather than a static subject. It traces the general trends in the progress of geometry with regard to the purpose of the course, its content and the method of teaching it not only in this country, but in Europe.

A convenient summary of deliberations of committees, magazine articles, and books on geometry. Traces the changes in European practice (pp. 25–28) in recent years; for example, Klein's summary of the demands in Germany for (1) a psychologic point of view; (2) better choice of material; (3) closer touch with practical applications; (4) the fusion of plane and solid geometry, and of arithmetic and geometry. "Intuitive geometry in plane and solid figures is started in the sixth or seventh school year. This is followed by algebra and geometry taught in parallel courses over a period of several years, the rigor increasing gradually from year to year. The two subjects are not fused into one course, but are kept distinct. This plan gives the teacher an opportunity to relate the subjects together without merging them, and it gives the pupil an appreciation of each subject by itself and also in relation to the

other subject. In the upper grades of the secondary school the elements of projective geometry and of descriptive geometry are introduced." In France similarly the work in demonstrative geometry runs parallel with arithmetic and algebra over a period of four years. On page 61 he gives Nunn's outline of a course in intuitive geometry for grades 2 to 7 inclusive, introducing a small amount of simple deduction in the 7th grade based on inductions from measurements and observation. Quotes two national committees in this country (p. 85) in favor of taking as undefined the terms point, line, surface, space, angle, straight line, curve, magnitude, plane, direction, distance, solid. Notes that Nunn's Principle of Similarity is a restatement of Wallis's proposal of 300 years ago to replace Euclid's Parallel Postulate with a Postulate of Similarity, as follows: "Given any figure, there exists a figure similar to it of any size we please." Quotes Carson (p. 102): "Let the statements of theorems which are easily acceptable to pupils be adopted as postulates, and let deductions be made from them with full rigor." Quotes John R. Clark and David Eugene Smith (p. 181) to the effect that *demonstrative* geometry cannot be merged with algebra to any large extent without losing its essential character. Recommends (p. 198) "a great deal of informal solid geometry in connection with the course in intuitive geometry."

Clark, J. R. *Mathematics in the Junior High School.* Gazette Press, 1925.

"In itself the transfer value of mathematics (or of any other subject) is insufficient to justify its being required of all pupils."

"The amount of transfer from mathematics depends upon the *method* of teaching the subject."

"Meanings, methods of attack, and attitudes are more transferable than skills and information."

Smith and Reeve. *The Teaching of Junior High School Mathematics.* Ginn, 1927.

Intuitive geometry considers the shape, size, and position of objects. (p. 138)

"In the intuitive stage of geometry there should be no dividing line between the study of plane figures and of solids." (p. 147)

We do not teach demonstrative geometry because we need it in practical problems of mensuration or because we thereby develop

space perception. These objectives are better obtained by means of intuitive geometry. "The real purpose of (demonstrative geometry) is suggested by the word 'demonstrative' rather than by 'geometry'. Nowhere else in his elementary education does the pupil come in close contact with logical proof. The chief purpose of this part of mathematics, then, is to lead him to understand what it is to demonstrate something, to prove a statement logically. He sees a sequence of theorems built up into a logical system." (p. 229)

"The results of geometric teaching can be improved by decreasing the number of theorems we teach, by concentrating on those propositions which are unquestionably fundamental, by recognizing the importance of original work, and by emphasizing the ability to demonstrate rather than the number of propositions studied. The legitimate claims of geometry as a method rather than as a body of facts can be met in less time than was formerly allowed." (p. 233)

Lietzmann, W. *Methodik des mathematischen Unterrichts.* I Teil. Quelle und Meyer, Leipzig, 1926.

This volume includes a general discussion of the teaching of mathematics: aims and purposes, practical suggestions for classroom practice, intellectual and practical preparation and continued study for a teacher of mathematics, helps for learning and teaching.

Chapter I is entitled Fundamental Theorems and Concepts of Geometry. In a discussion of definitions he suggests that much can be said about their choice, number and content; that the idea that some fundamental concepts cannot be defined appears too late in elementary geometries.

The Germans are accustomed to consider Euclidean and non-Euclidean geometry side by side and many are surprised at earlier objections to this procedure. The change dates back to Hilbert's "Grundlagen der Geometrie" which contributed a system of axioms complete in themselves and showed the value of the construction of geometries contrary to a particular axiom. A modern demand is that no necessary fundamental statements be lacking, as they are in Euclid, and requires that axioms be independent and not redundant. Mention is made of non-Euclidean geometries of Riemann and of Lobachevsky.

Lietzmann, W. *Methodik des mathematischen Unterrichts.* II Teil. Quelle und Meyer, Leipzig, 1923.

The introduction to geometry is apparently made in the Volksschule. He recommends a fusion of plane and solid geometry and that the teacher use no textbook, as texts for this work have not been successful, i.e. they have not gone into second editions. There should be models of solids which the child should see and feel and later identify in nature; the child must enlarge his vocabulary and learn to express himself orally and also in writing. Instruction should include the construction of models by pupils making their own patterns; skill in measurement as also in judging distances checked by measurement (also considering illusion in lengths); measuring and computing areas and volumes; similar figures; drawing, including simple construction problems.

Definitions should be more nearly discussion leading to an intimate knowledge; existence proofs are out of place for a child. Axioms should rest on intuitive work—if one discusses the independence of axioms it is necessary to discuss other geometries that differ from the Euclidean.

The aim of geometry is not the proof but the ability to prove. Indirect proofs should be used, but not too often.

There is a tendency for children to forget about space in a constant study of the plane only—a point in favor of fusion, which he believes in in moderation.

Similarity is treated after areas in Germany. The arithmetic theory of proportion should not be ignored and more use should be made of the factor of proportionality. He suggests treating harmonic division, golden section. The child should know other curves than the circle before leaving grade X.

Projective geometry will have an influence on the Euclidean or metric geometry in the lower grades. We are at present engaged in this change but have by no means completed the work, nor can one tell when it will be completed.

The present situation in geometry is unsatisfactory, probably for three reasons—didactic, psychological, and subject matter. The didactic difficulty may be overcome by enlarging the conception of the geometrical problem. The psychological difficulty became apparent as a result of stricter treatment of axioms and must be met by an approach compatible with the development of the child. The subject matter difficulty can be met by treating the old geometry from the point of view of the new geometry.

Meeting of parallel lines at infinity should not be introduced in the seventh grade because it is beyond the comprehension of the pupil; it might also well be ignored later in connection with harmonic division.

There is also doubt in the mind of the author of the propriety of teaching duality, although the relation of pole and polar, of Pascal's and Brianchon's theorems will escape few teachers.

It is the purpose of geometry not to exhaust work on any one figure but to teach congruence, areas, similarity, etc.

With respect to texts, he considers motion (rotation, translation, etc.) rather generally accepted. Hubert Muller and Henrici and Treutlein have probably gone farther in their texts than most along this line but they have not been successful because they shot beyond their mark. He believes that Henrici and Treutlein is a step in the right direction.

Lietzmann, W. *Aufbau und Grundlage der Mathematik.* Teubner, 1927.

A book for teachers on the foundations of mathematics. There is a chapter on logic in mathematics which discusses fundamental conceptions and their relation to each other; exactness of definitions and mistakes in formulating them; a discussion of theorems, converses and opposites, direct and indirect conclusions; inductive and deductive methods; proofs; necessary and sufficient conditions; proofs of impossibility; multiplicity of solutions.

Chapter II treats of the foundations of geometry. Among other things he discusses the congruence axioms and freedom in the selection of axioms. He lists the systems of Hilbert, Veblen, and Pieri. He suggests that the fundamental principle might be rigidity, translation, rotation, or even symmetry.

Simon, Max. *Ueber die Entwickelung der Elementargeometrie im XIX Jahrhundert.* Teubner, 1906.

A report of the development of geometry in the 19th century with a detailed list of leaders in teaching and the most important books and articles on various phases of geometry—printed in all countries, i.e., France, Germany, England, United States, Italy, and other countries.

On Method. The method of teaching geometry is closely related to philosophy, the question being whether it is pure or applied mathematics. All countries have magazines primarily interested in method and in most countries there are also associations for the

purpose of furthering good teaching. In England the Mathematical Association under the influence of DeMorgan and Sylvester have been working away from the rigid acceptance of Euclid to a syllabus, a new and better organized plan, which the author believes had not been realized by 1900.

The 19th century produced a movement in geometry away from the dogmatic point of view. Problems and constructions appeared, and more importance was given to intuitive geometry. The introduction of more intuitive geometry tended toward a fusion of plane and solid geometry. Further development of non-Euclidean geometry, and criticism of the fundamentals, emphasized the independence of the parallel axiom and reaffirmed Euclid's mathematical insight. (Todhunter, Sannia, D'Ovidio, Faifofer, and Max Simon.) There is also a greater tendency toward meeting utilitarian needs. The volume contains a discussion of major problems in geometry, such as parallel axiom, quadrature, etc.

Becker, J. Karl. *Zur Reform des geometrischen Unterrichts.* Wertheim, 1880.

Becker indicates that attempts have been made since 1800 to free geometry from the rigid form into which Euclid forced it. That if a teacher does not obtain results it is due either to the fact that he has not made use of the progress and improvements that have been made, or else to reasons not connected with methods. That success in the teaching of geometry more than in many other subjects depends upon the personality of the teacher. That it is possible to do for the geometry of "mass and form" that which Steiner did for projective geometry. He continues to show how he has attempted to do this in his textbook despite the contrary criticism of one Herr Erler. He also sets forth the fundamental principles used in his book among which he includes the parallel axiom and the use of an axis of symmetry. He does not include the congruence statements in these but apparently develops them.

According to his own account his book varies most from other texts of his time in its treatment of quadrilaterals based on work with circles.

Pieri, Hilbert, Halsted, Veblen, Heath, Enriques.

Pieri in 1899 formulated a set of assumptions for metric geometry based on the undefined terms "point" and "rigid motion." (See

J. W. Young's Fundamental Concepts of Algebra and Geometry. Macmillan, 1911)

Hilbert in his *Foundations of Geometry* (Open Court, 1902) lays down axioms for a geometry based on the ideas "connection," "order," "parallel," "congruent" in which "point," "line," "plane," "between," "congruent," . . . are taken as undefined. Equality is then defined in terms of congruence. The last of his assumptions under congruence amounts practically to assuming that if two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, the remaining sides and angles are respectively equal. His treatment makes it unnecessary to mention rigid motion and superposition. He does not show us in detail what sort of geometry would result from his assumptions, but his ideas are reflected in the British Report on the Teaching of Geometry in Schools and have influenced the teaching of demonstrative geometry in other countries.

Halsted's *Rational Geometry* (Wiley, 1904) is an elaboration of Hilbert's Foundations into a school text. The treatment avoids explicit mention of superposition and incommensurables. These gains are more than offset by the pedagogical difficulty involved in long proofs of the obvious.

In an article in the Monographs on Modern Mathematics compiled by J. W. A. Young (Longmans, 1911) Veblen takes "point" and "order" as undefined and establishes postulates for geometry in terms of them. His ideas are not developed into a text for school use, and probably could not be so developed without considerable modification; but they offer striking illustration of a geometry with "as few assumptions as possible," and can be used therefore to support the movement for a broader range of assumptions in elementary geometry. For since our usual assumptions can no longer be regarded as a minimum list, and since the absolute mathematical minimum is so forbidding pedagogically, we can consider ourselves free to compromise the mathematical and pedagogical claims.

The store of information contained in Heath's *Thirteen Books of Euclid's Elements* (Cambridge University Press, 1908), in Enriques's *Questioni riguardanti le matematiche elementari* (Zanichelli, 1900, 1914, 1925), and in Enriques's *Gli Elementi d'Euclide e la critica antica e moderna* (Stock, 1925) is too great to be compassed here in any brief review, but the committee does not wish to appear

unmindful of the important contributions of these men to the teaching of elementary geometry.

III. Textbooks

Spencer, W. G. *Inventional Geometry.* American Book Company, 1876.

A series of 446 skillfully graded problems for the pupil to solve without assistance, principally by means of constructions which he is led to devise for himself out of his solutions to preceding problems.

Nichols, E. H. *Elementary and Constructional Geometry.* Longmans, Green, 1902.

Designed for pupils of age 12 or younger. A series of 678 questions and exercises in plane and solid geometry through which the pupil becomes thoroughly conversant with the facts of geometry.

Campbell, W. T. *Observational Geometry.* American Book Company, 1899.

The author's object was to train the child's powers of observation by leading him to consider simple geometric forms and relations in the spirit of the experimental laboratory. In the first half of the course the pupil constructs models of many solids out of cardboard and learns relations in plane geometry only incidentally. The second half of the course develops the facts of plane geometry through constructions, measurements, computations.

Failor, I. N. *Inventional Geometry.* Century, 1904.

An adaptation of Spencer's Inventional Geometry, with diagrams.

Hedrick, E. R. *Constructive Geometry.* Macmillan, 1917.

The author's purpose is to acquaint students with the elementary geometric forms and constructions, so that those who go on to study formal demonstrative geometry will not be distracted later by having to learn fact and logic simultaneously, and so that those who will never go on to the formal study may acquire at least the valuable factual material. A notebook with blank pages for the pupil's constructions.

Smith, E. R. *Plane Geometry developed by the Syllabus Method.* American Book Company, 1909.

The book begins with a brief chapter on logical principles, discussing converse, obverse, and contrapositive propositions. There is no discussion of geometric constructions as commonly given in the introductory chapter of most texts today.

The author proves a few theorems to show what is expected, and then relies for the most part on his analysis of the theorem to suggest the necessary hints, or better, to elicit them from the pupils.

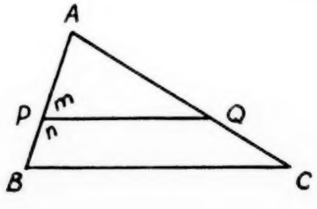
Betz and Webb. *Plane Geometry.* Ginn, 1912.

The authors invert the usual order of Books III and IV, treating area before proportion. Their treatment of area in Book III assumes as a "Fundamental Principle" that the area of a rectangle is equal to the product of its base and altitude. This involves implicitly the incommensurable case and obviates later mention of incommensurables in connection with proportion. At the beginning of each Book there is a summary of the definitions and assumptions which will be used. The earlier theorems under each topic are proved in detail; but as the student advances proofs are either omitted, or given in outline, or are suggested by an analysis. There is brief treatment of axial symmetry, coordinates, Simpson's Rule, trigonometric ratios, and at the end of the book a careful treatment of limits.

Young, J. W. and Schwartz, A. J. *Plane Geometry.* Holt, 1915.

Makes great use of drawing and motion. Uses rotation and symmetry to prove two lines parallel if alternate interior angles are equal.

Postpones congruent triangle theorems till after parallels and sum of angles of a triangle have been considered. Treats incommensurable cases in a very simple yet rigorous manner by means of inequalities. For example, both $\frac{PB}{AP}$ and $\frac{QC}{AQ}$ lie be-



tween $\frac{n}{m}$ and $\frac{n+1}{m}$ and so differ by less than $\frac{1}{m}$. This difference can be made less than any assignable positive number. Introduces trigonometry of the right triangle after similar triangles. The laws of sines and cosines for oblique triangles are given in a later chapter.

Schutts, G. C. *Plane and Solid Geometry* (*Suggestive Method*). Atkinson Metzger Co., 1916.

Uses conventional material in geometry including the trigonometric relationships under proportion. Direct method is used for proof of equality of alternate interior angles of parallel lines. Proofs are suggested, sometimes rather fully. Chapter VI includes treatment of incommensurable magnitudes, variables and limits.

Reeve, W. D. *General Mathematics, Book II* (*principally Geometry*). Ginn, 1922.

The introductory chapter contains, besides many constructions, a definition of parallel lines as lines which make equal angles with a transversal; also, formal proofs of theorems on parallels and the sum of the angles of a triangle.

In the chapter on congruence there is mention of duality. Then polygons and prisms. Before proceeding to consider circles the author inserts a chapter on *lines and planes in space* which includes most of the material in "Book VI" of Solid Geometry. The chapter on circles includes further pertinent reference to solid geometry.

The author states in the preface, "Moreover, many of the propositions of solid geometry are analogous to those of plane geometry and can be easily taught in connection with them." This is supported by the further declaration that "the material included has been given a careful trial by the author in the classroom and constitutes what has proved to be a thoroughly usable course in general mathematics for the second year, or tenth grade, of the high school."

Smith, R. R. *Beginners' Geometry*. Macmillan, 1925.

Assumes the first two theorems on congruent triangles. These are proved later in Chapter IV.

Assumes in addition to the usual parallel postulate the propositions concerning the unique perpendicular to a line at an internal

point, and through an external point. Assumes perpendicular to be shortest distance from point to line.

Assumes theorem concerning hypotenuse and acute angle of two right triangles, later proved in Chapter IV.

McCormack, J. P. *Plane and Solid Geometry.* Appleton, 1928.

After a brief introduction come the usual three propositions on congruent triangles, then the construction of the perpendicular bisector and its treatment (in all but name) as a locus; parallel lines, angle-sum, the converse of the isosceles triangle theorem, and congruent right triangles. Constructions in Book I are given right after the theorems to which they are most closely related instead of being grouped together.

Two of the usual theorems on similar triangles appear early in Book III; the third comes at the very end, after an extensive treatment of numerical trigonometry, including the Law of Sines for acute angled triangles. The author says (p. 234) that this trigonometry belongs logically in a course in plane geometry.

In dealing with areas the author assumes that the area of a rectangle is the product of its base and altitude. He considers rectangles with incommensurable sides and leads the student to infer the reasonableness of this assumption.

"Investigation problems," preceding each important theorem, lead the pupil to discover the proposition and to work out the proof for it.

Morgan, Foberg, Breckenridge. *Plane Geometry.* Houghton Mifflin, 1931.

A brief introductory chapter lists the usual definitions and postulates and gives exercises based on these. It sets forth the desired form of demonstration—Given—To prove—Analysis—Proof—using the theorem concerning vertical angles as a sample. A second sample is the theorem which states that two right triangles are congruent if the hypotenuse and an acute angle of the first are equal respectively to the hypotenuse and an acute angle of the second. The work in geometric constructions recommended for Grades 7 and 8 and often found in the introductory chapter of geometries is in this book included in the body of the text.

The usual division into five books is replaced by a division into twenty-four topics, Congruent Triangles, Parallel Lines, . . . Tangents and Secants, the Measurement of Angles in Circles, . . . in

the usual order. The strategic location of the theorem concerning congruent right triangles in the introduction as a sample demonstration makes it available for the proof of the theorem concerning alternate interior angles of parallel lines, although the topic Congruent Right Triangles comes after Parallels.

The construction problems are exemplified by sequences of diagrams exemplifying the separate stages of the construction by the "See-It-Grow" method.

Attention is called to incommensurable cases, but proofs are omitted. Harmonic division, extreme and mean ratio, the regular decagon, and the trigonometry of the right triangle are included in an Appendix. The usual index is followed by a special index of all the exercises in the book arranged by topics.

Sykes, Comstock, Austin. *Plane Geometry*. Rand McNally, 1932.

The material is divided into units of instruction closely paralleling the usual order of topics. An important exception is the treatment of similar triangles, which is split into two parts. The first follows upon the treatment of the circle and proceeds with the usual discussion of proportional segments, with an optional section on trigonometric ratios. After this comes area, with the incommensurable case mentioned but the proof withheld. The second part of the material on similar triangles and polygons is now taken up, leading to regular polygons and the mensuration of the circle.

The introduction omits most of the geometric drawing so commonly recommended for the junior high school but generally appearing also in the earlier pages of texts devoted to demonstrative geometry. Preceding the proof of each "book theorem" is a section entitled "analysis and construction" which shows the general plan of the proof.

Cowley, Elizabeth B. *Plane Geometry*. Silver, Burdett, 1932.

A relatively long introduction exhibits many artistic diagrams, lists the usual definitions and postulates, sets forth the elementary constructions of geometry, and studies the structure of a theorem, using as example the theorem, "If two parallel lines are cut by a transversal, the alternate interior angles are equal." The proof of this depends upon the definition, "If two lines can be cut by a transversal so that a pair of corresponding angles are equal, the lines are called *parallel lines*." It depends also upon the assump-

tion—not uncommon in recent French and German texts—that if a line intersects a transversal at P, then at any other point Q of the transversal there is *only one angle* "corresponding" to the angle at P, and consequently *only one parallel* to the first line.

Book I goes on at once into further work with parallels, the proofs of the congruence theorems being concluded in the Appendix. From then on the theorems are ordered in the five books according to common practice. The discussion of each theorem yields an analysis, or else a plan, of the proof. A discussion of trigonometry that will be of interest to teachers will be found in Book III.

Smith, Reeve, Morss. *Text and Tests in Plane Geometry.* Ginn, 1933.

This large paper covered book of 286 pages is a combined textbook, student's workbook, and "testbook." There are two introductory chapters, or "units." The first, of 34 pages, is called Informal Geometry. It deals with geometry in nature, art, and architecture, gives practice in geometric drawing, and leads the pupil to experimental inquiry concerning two propositions on congruent triangles, withholding till much later a rigorous demonstration of these congruence theorems.

The second introductory unit, of 36 pages, is called an Approach to Demonstration. It sets forth the need for proving statements; discusses definitions, axioms, and postulates as "bases of proof"; and employs the proposition concerning vertical angles to exhibit the form of a demonstration. It continues with propositions concerning parallels, perpendiculars, and the sum of the angles of a triangle in order to show a "geometric chain of truths." The definition "Two lines in a plane which have the same amount and direction of rotation from a third line are parallel" makes it possible to treat the alternate-interior angle case without employing congruent right triangles.

The third unit, entitled Congruence, makes free use of superposition in proving the usual theorems on congruent triangles and isosceles triangles. The proposition concerning right triangles which have hypotenuse and a side respectively equal is dissected in order to trace its "ancestry," to exhibit its "family tree." Each step in the proof is shown to be a definition, a postulate, or an earlier proposition the proof of which depends on definitions, postulates, and still earlier propositions, until the entire proof of

the given proposition is related to all the definitions and postulates on which it rests.

The remaining units deal with Parallelograms, Loci, Circles, Inequalities, Proportion and Similarity, Ratio in Trigonometry, Areas of Polygons, Circle Measurement.

Gilmore, Roberta I. *Geometry Originals based on Concepts from Physics.* Term paper prepared for a graduate course in secondary education at the University of Pennsylvania, 1931.

This compilation shows that most of the concepts of elementary geometry can be exemplified by situations drawn from elementary physics.

Birkhoff and Beatley. *Geometry.* Experimental Edition (Harvard University), 1933.

The introductory chapter considers the relation between reasoning in geometric situations and reasoning in everyday life. It emphasizes the need of undefined terms, definitions, and assumptions in every logical system. It shows the possibility of altering the original assumptions and studies the effect of so doing.

The main body of the book presents a compact treatment of geometry based on five assumptions. One of these serves to link the system of real numbers with points on a line, with the result that the distinction between commensurable and incommensurable lengths can be ignored whenever desirable. A second assumption performs the same service with respect to the measurement of angles. A third asserts that if two triangles have an angle of one equal to an angle of the other and the including sides proportional, the triangles are similar. From this it is possible to go in a few steps to the proof of the theorem concerning the sum of the angles of a triangle, the Pythagorean theorem, and the other theorems concerning similarity and parallelism. Congruence is treated always as a special case of similarity in which the factor of proportionality is 1.

The final chapter returns to a consideration of the logical structure of geometry and its application to situations outside geometry and mathematics.

Blackhurst, J. H. *Humanized Geometry.* Second experimental edition (Drake University), 1934.

The sub-title of this book, An Introduction to Thinking, indicates the author's intent to use geometry as an aid in "studying

the ways in which effective thinking is done." The usual introductory pages of definitions and drawings are omitted. The text begins at once with a comparison of induction and deduction based on a study of the angles formed by two intersecting lines. In the course of a deduction the need of certain axioms appears, leading to the discussion of assumptions. Definitions and the analytic method are considered; then the syllogism and its major and minor premises (pp. 83 and 108); and then the indirect method.

The main body of the book covers less than the usual content of plane geometry, but makes good this deficiency by including the omitted material in a final chapter "for superior students and those preparing to pass a college entrance examination." Throughout the book there are extended historical notes.

Henrici, O. *Congruent Figures*. Longmans, Green, 1891. (Out of print.)

Gives comprehension of geometric facts intuitively, but discusses first principles rigorously and with conscious emphasis on logic.

Godfrey and Siddons. *A Shorter Geometry*. Cambridge University Press, 1921.

First 74 pages identical with Geometry for Beginners. Treatment in accordance with explanation in "The Teaching of Elementary Mathematics" by the same authors.

Carson, G. St. L. and Smith, D. E. *Plane Geometry*. Ginn, London, 1915.

The introductory section of 90 pages is intuitive geometry of two and three dimensions with a little numerical trigonometry.

In the formal treatment of geometry which follows many obvious propositions are assumed as postulates for a first reading; the proofs are indicated in smaller type for the second reading. Propositions to be thus postulated for the "first reading" are: Vertical angles are equal; base angles of an isosceles triangle are equal; only one perpendicular to a line from an external point; oblique lines from a point in a perpendicular which cut off equal segments are equal; of two oblique lines cutting off unequal segments, the more remote is the greater; the perpendicular is shortest distance from point to line; two lines perpendicular to same line are parallel; through a given point there can be only one parallel to a given line; a line perpendicular to one of two parallels is perpendicular to the other; if two parallels are cut by a transversal, alternate

angles are equal, exterior-interior angles are equal, interior angles supplementary (the authors waver on the question of postulating these propositions on the transversal); two lines parallel to third line are parallel; sum of two sides of a triangle greater than third side. The final postulate of Book I (tacitly assumed until then) establishes rigid motions as fundamental to proofs by superposition.

Areas and the Pythagorean Theorem are treated in Book II ahead of circle, similar triangles and proportion.

The following propositions are proved in Book III (circles): Lines can meet a circle in no point, in one point, or in two points, but no line can meet a circle in three points; the locus of a point from which tangents to two given circles are equal is a line (radical axis) perpendicular to line of centers.

In Book IV proportion is treated by means of inequalities after the manner of Euclid. In Book V, on similar triangles, the theorems on congruent triangles in Book I are referred to as special cases under similar triangles. There is also a theorem on the *center of similitude* of similar triangles similarly situated.

Discussion of the constant ratio of circumference to diameter is reserved for the appendix, together with the approximate evaluation of π, and the development of the formula πr^2 for the area of a circle.

The appendix lists also the 11 axioms and 24 or more postulates (including theorems to be taken as postulates in the first attack on formal demonstrative geometry).

Hall and Stevens. *A School Geometry*. Macmillan, London, 1928.

Except for the inclusion of certain advanced topics, the ground covered in the first five books closely resembles recent practice in the United States. The arrangement of the material is closer to the Euclidean tradition, however. Following Part I, which resembles our Book I, the second "Part" deals with areas. Part III is concerned with circles, but includes material on regular polygons which our books usually give later in Book V. The nine-point circle receives mention at this point. Part IV is entitled Geometrical Equivalents of Some Algebraic Formulas, based on Euclid's Book II and ordinarily not mentioned in our texts. Part V deals with similar triangles and proportion, ending with Ptolemy's Theorem, maxima and minima, harmonic section, center of similitude, poles and

polars, radical axis, inversion, Ceva's Theorem, and Menelaus's Theorem.

Durell, C. V. *Elementary Geometry.* G. Bell and Sons, London, 1931.

This book divides instruction in geometry into the three stages A, B, C recommended by the British Report on the Teaching of Geometry in Schools.

Stage A, of only 16 pages, is a brief informal discussion of definitions with some reference to measurement and drawing.

Stage B, comprising 36 pages, gives informal demonstrations of propositions concerning parallel lines and the sum of the angles of a triangle; sets elementary exercises based on these; assumes the three propositions on congruent triangles; and gives a brief treatment of the trigonometry of the right triangle. Except for this small bit of trigonometry Stages A and B together are comparable to what one finds in the introductory chapter of geometries in the United States.

The bulk of the book is devoted to Stage C, the deductive development of congruence, parallelism, and so forth. The influence of Euclid is revealed in the section on areas which precedes the treatment of the circle; also in the insertion of a section on regular polygons between The Circle and Similar Figures.

The appendix contains a formal treatment of the propositions on perpendiculars, parallels, and congruence, covered informally in Stage B.

Méray, C. *Nouveaux Éléments de Géométrie.* Dijon, 1874; 1903.

Emphasizes motion and frequent reference to solid geometry. Begins with the motion of translation applied to parallel lines and planes; then perpendicular lines and planes. Uses rotation to introduce plane and dihedral angles. Angle-sum, similar triangles, the Pythagorean Theorem, loci in two and three dimensions, areas, volumes of solids, similitude come next. The familiar properties of the circle necessary for the simplest constructions are used intuitively in the foregoing where desired, but there is no formal treatment of the circle until rectilinear figures have been dealt with at length. Finally the formal treatment of circle, inversion, regular polygons, sphere, cylinder, and cone.

Borel, E. *Elements of Geometry*. Paris, about 1909.

Attempts to fuse plane and solid geometry. Emphasizes continuity and makes free use of algebra and trigonometry.

An exposition of the properties of space rather than of deductive logic.

Hadamard, J. *Géométrie Plane.* Colin, 1911; 1922.

Designed for mature pupils in secondary school, this purports to be a logically rigorous presentation of geometry and so ignores the intuitive approach so well adapted to earlier stages of instruction. Though claiming logical accuracy, it sees no reason for continuing the unnecessarily academic and complex treatment customarily accorded to angle in Book I. Angle therefore is linked intimately with the circular arc from the beginning: angles are proportional to their arcs; and the uniqueness of the perpendicular at a point in a line is made to depend upon the uniqueness of the mid-point of a semicircular arc.

The earlier propositions are arranged as follows: base angles of isosceles triangle; bisector of vertex angle of isosceles triangle is perpendicular bisector of the base (both of these proved by overturning the triangle); a.s.a.; s.a.s. (both by superposition); s.s.s. by reference to perpendicular bisector of base of isosceles triangle.

Following similar triangles is an extended treatment of homothetic triangles, center of similitude, anharmonic ratio, harmonic ranges and pencils, poles and polars, and inversion.

The appendix carries an interesting note on area to show how area can be arrived at without taking the idea for granted in advance. For example, in every triangle the product of base times altitude is constant whichever side be chosen as the base; this follows at once from a consideration of similar triangles. The area of the triangle is then defined as the product of the two numbers, length of base, length of altitude, times a constant K. Later developments lead one to desire K to have such a value that the area of the unit square will be 1: the corresponding value for K is $\frac{1}{2}$.

The author is not tempted to follow Méray in fusing plane and solid geometry. Two matters as difficult as *space perception and pure logic* ought, he says, to be treated separately. (p. vi, footnote.)

Chenevier et Desbrosses. *Géométrie.* Hachette, 1922.

These authors present the theorems of geometry as a series of

problems for discussion and investigation. In each case the formal enunciation of the theorem is the final step, rather than the first. There is constant appeal to intuition, but intuition is not allowed to take the place of proof. For example, lines drawn by means of a tri-square sliding along a straightedge obviously cannot meet; this is proved, however, by supposing that they do meet and showing that a perpendicular dropped from the point of intersection to the straightedge would have to bisect two different line segments of a given line having common endpoint. The order of the theorems departs widely from Euclid's order. Before taking up similar triangles and proportion there is extended treatment of constructions, including orthogonal projections of three dimensional figures.

Chenevier. *Précis de Géométrie Plane* (*classes de 4^e^, 3^e^*). Hachette, 1925.

Emphasizes an experimental procedure leading naturally to the enunciation of each theorem, which is then proved in the conventional manner, though softened a bit to make it appropriate for the equivalent of our 9th and 10th grades. Two lines drawn by tri-square sliding along a straightedge apparently can never meet; this is proved by the indirect method, showing in this case that if the lines met once they would have to meet in a second point. The elementary theorems concerning chords and arcs of a circle follow immediately upon parallels and perpendiculars. Then come the construction of triangles according to various criteria and the recognition (but no further proof) of the equality of triangles under the usual three conditions. Then parallelograms, the angles inscribed in a circle, similar triangles, and regular polygons, postponing area until the end.

Chenevier. *Cours de Géométrie* (*classes de 2^e^, 1^re^*). Hachette, 1927.

This text contains a more rigorous treatment of the geometry studied in the two previous years and presented in the author's Précis de Géométrie Plane. Nowhere, even in this more rigorous treatment, do we find the *Given-To Prove-Proof* arrangement of our texts. Instead, the presentation is as follows: *Problem-To study* the intersection of two planes each perpendicular to a third plane (or, *To compare* an inscribed angle with the corresponding central angle; or, *To see* if two polygons with angles respectively equal and with corresponding sides proportional are similar).

Then a paragraph giving the proof of the theorem, which is not definitely stated until the very end. The "problem" practically states the theorem to be proved; its enunciation at the end is equivalent to a Q.E.D. The tenor of the presentation encourages the spirit of inquiry, but does not insure it. The proof itself is set forth in a simple direct paragraph which is rhetorically far superior to our stilted method, symbolized by the line drawn down the middle of the page.

Bourlet. *Cours abrégé de Géométrie.* Hachette, 1928.

A treatment of geometry based on drawing and motion. Constructions can define geometric elements and suggest propositions to be proved. Translation and rotation are fundamental methods of proof. The result is a geometry quite different from Euclid's and depending in a fundamental way on intuition. For example, two lines in the same plane are called parallel if one is carried into the other by a translation. That there is one and only one line through P parallel to l is established by sliding a tri-square along a straight-edge. A geometry so constructed naturally develops the properties of parallel lines and transversals early, and the sum of the angles of a triangle. Axial symmetry defined in terms of a folding of a half plane leads to perpendicular bisectors, bisectors of angles, the isosceles triangle, elementary loci, and simple constructions. Not until after the usual theorems on the circle (our Book II) do we come to the theorems on equal triangles. There is less of novelty in the treatment of similar triangles, regular polygons, and areas.

Becker, Johann Karl. *Die Elemente der Geometrie auf neuer Grundlage streng deduktiv dargestellt.* Berlin, 1877.

For teachers, not for school use (p. vi). If there are actually propositions which are undeniably obvious, by what right do we try to minimize the number of them? Why not rather rely on the obvious as far as possible and resort to deduction only when other means fail? He develops certain fundamental concepts of plane and solid geometry concurrently. For example, he treats of the sphere first, then the circle, then the cone. Of the congruence theorems, "s.s.s." comes first. The plane is defined as that particular cone with elements perpendicular to the axis. He proceeds then to a systematic treatment of the theorems of plane geometry, followed by solid geometry.

Henrici, J. und Treutlein, P. *Lehrbuch der Elementar-Geometrie*, I Teil, 3 Auflage. Leipzig, 1897.

In the first part, the order of the theorems is dependent upon the method used for their proof. First fundamental notions concerning lines and angles are established, including the fact that only one perpendicular can be drawn from a point to a line. Then using symmetry with respect to a point, the authors continue the study of lines and angles, parallels, triangles and trapezoids, also the angle-sum theorems. Next, they introduce symmetry with respect to a line and on this basis study perpendicular bisectors, the isosceles theorems, etc. In the next chapter they introduce translation and rotation, prove the congruence theorems, and consider triangles having only a few parts equal.

The third part is concerned with circles and constructions; and the fourth part with polygons, beginning with a discussion of the quadrangle and quadrilateral with respect to an axis of symmetry and a center of symmetry. The book closes with the work on areas. In addition there are forty-four pages of exercises to be used with the chapters often as foundation material. The authors feel that this book makes more use of protractor and ruler than is usually the case.

Proofs are much more informal than in our texts.

Henrici, J. und Treutlein, P. *Lehrbuch der Elementar-Geometrie*, III Teil, 2 Auflage. Leipzig, 1901.

This is a continuation of the first book, considering the material of solid and spherical geometry and synthetic projective geometry. There is free use of central symmetry, axial symmetry, and symmetry with respect to a plane.

The work on projection includes orthographic and stereographic projection, and presents the conic sections as an important application of projective methods. Trigonometry is used whenever necessary.

Again about fifty-five pages of exercises are added to aid in the treatment of the subject matter. The proofs are more nearly discussions than proofs in our sense of the word.

Mahler, G. *Ebene Geometrie*. Leipzig, 1906.

No formal statement of axioms; no lists of definitions. Axioms are assumed tacitly throughout. Great reliance is placed on the student's intuition. Motion and symmetry are freely used.

Behrendsen, D. und Götting, E. *Lehrbuch der Mathematik nach modernen Grundsätzen* (für höhere Mädchenlehranstalten etc.) Leipzig und Berlin, 1909.

This edition is designed for girls schools. It is an adaptation of one originally planned for boys. The author claims that it closely follows the recommendations of Professor Klein. A deductive procedure based on the axiomatic foundation to be found in the Euclidean geometry has been avoided. The first geometric notions are built up by training the eye and the hand to estimate measurement; by using the straightedge, compasses, ruler, and protractor. Later, simple deductive reasoning is introduced. A chapter on space relationship, which does not occur in the book for boys, has been added but the treatment is of an introductory character only.

Functionality is introduced wherever possible and is particularly evident in the portion of the book devoted to algebra. In geometry it is used in the work on areas and proportions.

Symmetry with respect to an axis is taught.

The exercises are found adjoining the material upon which they are based.

Malsch-Maey-Schwerdt. *Zahl und Raum*. Quelle und Meyer, Leipzig, 1927.

A continuous course passing gradually from the intuitive stage—including solids—to the logical demonstrative stage, though never attaining the formalism common to the latter stage in England. For example, parallels are introduced as follows (III Heft, Geom. I, p. 24). The authors slide a draftsman's triangle along a straightedge, drawing lines for each position of the triangle. They then say, "We observe from the drawing that

1) through P there is only one parallel to l; and

2) if two lines are met by a third so that corresponding angles are equal, the lines are parallel." Here are two important assumptions, not specifically noted as such however. The authors proceed next to prove (by the Indirect Method) the proposition that if two lines are parallel, corresponding angles are equal; and later theorems. Such a development gives gradual insight and practice in the details of ordinary deductive logic but fails to disclose the larger aspects of a logical system.

An extensive treatment of trigonometry grows out of the work on similar triangles.

Lietzmann, *Geometrische Aufgabensammlung für Unterstufe;* and Lietzmann-Zühlke, *Geometrische Aufgabensammlung für Oberstufe.* Teubner, 1930.

This is a moderate fusion of plane and solid geometry in accordance with the official Prussian syllabus of 1925. The exercises in plane geometry are related often to three-dimensional situations, and the authors indicate the possibility of further reference to solids in connection with areas and similar figures. The instruction reflects the recent demand for greater emphasis on drawing and on descriptive geometry.

The exercises serve very often as an integral part of the development of a proposition; in fact, almost no proposition is demonstrated in full in the text. The object is to elicit pupil participation in the development of the entire subject. There is no attempt to demand the formality of statement which is so large a part of demonstrative geometry in the United States.

So far as possible the student is led to apply geometry to life about him; in fact, the authors prefer at this stage to minimize distinctions between mathematics and its applications, and encourage their pupils to contemplate situations which suggest problems leading to the development of mathematical principles. The boundaries between mathematics and the related sciences are purposely blurred for the same reason.

The topics are treated in the following order. For Unterstufe, first, an introductory section presenting geometric designs involving straight lines, polygons, circle and central angle. Then a succession of chapters dealing with parallel lines; the sum of the angles of a triangle, symmetric and congruent triangles; quadrilaterals; the circle; areas; similarity and proportion; a bit of descriptive geometry relating to lines and planes in space; trigonometry of right and oblique triangles; parallelepiped, prism, pyramid, cylinder, cone, and sphere. For Oberstufe, trigonometry completed; more solid geometry with much drawing; descriptive geometry; spherical trigonometry and the astronomical triangle; synthetic treatment of conics; analytic geometry.

Reidt, Friederich. *Die Elemente der Mathematik, Dritter Teil, Stereometrie.* Berlin, 1868.

Typical subject matter for solid geometry, theorems, proofs, corollaries with pages of applied exercises. On the theorem con-

cerning a line perpendicular to each of two intersecting lines there are three proofs, those of Euclid, Cauchy, and Legendre respectively. Regular polyhedra are given very thorough treatment, as are also the obelisk and the prismatoid.

Reidt, Wolff, Kerst. *Die Elemente der Mathematik, Vol. II, IV, Geometrie* (*Unterstufe and Oberstufe*). Berlin, 1932.

After an informal, inductive, approach the subject gradually becomes more and more deductive. Material from three dimensions is given at various pertinent places in the treatment of plane geometry. The method of experiment gradually gives way to logic in the geometry of two dimensions, but the experimental and inductive approach is then applied to related topics in three dimensions. The student first meets a "proof" on page 48 in connection with an "original" involving the equilateral triangle.

The arrangement of topics closely resembles that of Lietzmann-Zühlke, except that Reidt-Wolff-Kerst pay even more attention to perspective drawing and descriptive geometry, and include nothing at all of trigonometry. They treat trigonometry in Volumes I and III of this series in connection with Arithmetic, Algebra, and Analysis.

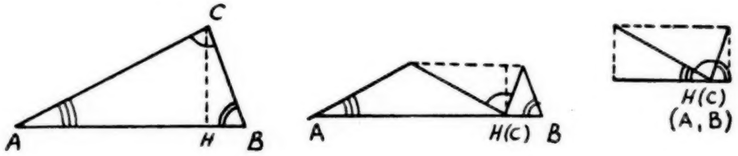
Ciamberlini, C. *Misurazione*. Paravia, 1923.

A booklet on the measurement of areas and volumes of polygons, circles, prisms, pyramids, cylinder, cone, and sphere; designed especially for students in industrial and commercial courses.

Enriques, F. e Amaldi, U. *Nozioni Intuitive di Geometria*. Zanichelli, 1928.

The plane is exemplified by a smooth sheet of paper. This folded and creased exemplifies a straight line. A pencil dot indicates a point. Half-line, line-segment, the sum, difference, product, quotient of line-segments are treated by direct appeal to common observation, intuition. Line-segments are added by copying them onto an improvised straightedge; cutting a sheet of paper along a crease shows that a plane is divided into two parts by a straight line; folding a crease onto itself reveals a second crease perpendicular to the first; parallels are developed by means of tri-squares; the sum of the angles of a triangle is demonstrated by paper-folding. In similar manner, by means of drawing instruments, scissors, and

pastepot the other essential facts of plane and solid geometry are imparted; frequent numerical exercises serve to fix these facts.



Enriques, F. e Amaldi, U. *Geometria Elementare, (piana)*. Zanichelli, 1929.

The authors postulate the possibility of copying geometric figures by means of instruments (i.e. by rigid motion). They postulate also the first proposition on equal triangles, calling it "the first criterion for the equality of triangles." By means of this they deduce next the isosceles triangle theorems, and then "the second and third criteria for the equality of triangles" (i.e. *a.s.a.* and *s.s.s.*). Their treatment of parallels follows Euclid in proving first those propositions which establish the criteria for parallelism in terms of supplementary or equal angles formed by a transversal; they base the converse propositions on Playfair's Postulate instead of Euclid's, however. The treatment is rigorous throughout.

Veronese, G. *Elemente di Geometria*. Draghi, 1910.

Although for students who will eventually be teachers of geometry a "rational deductive" treatment of geometry is more important than the experimental or inductive treatment common in the lower grades, an even better course for such students would be attained by softening the familiar logical presentation with an approach depending less on logical subtleties and appealing more to the students' intuitions. The transition from the experimental and observational type of geometry to this (somewhat) intuitional and purely deductive geometry comes about naturally enough. For occasionally out of a series of geometric relations established first of all experimentally and inductively there appears one which seems to be a necessary logical consequence of some of the others. This opens up the possibility of a rational deductive method by which from certain fundamental assumptions all the other geometric relations can be developed deductively without further recourse to verification by practical means.

The author recognizes the force of the students' intuition in framing his list of postulates, but is careful to distinguish those postulates which are pedagogically desirable from those which are mathematically necessary.

He begins by considering line and line segment as collections of points and defines sum, difference, product, quotient of line segments. Parallel lines are defined in terms of central symmetry. All the lines joining the points of a given line with a point outside the line determine a plane. All of these lines which lie between two given lines determine an angle. The line segment joining two points on opposite sides of a given line intersects the given line—a theorem. A point is *within a triangle* if it is within a line segment joining a vertex of the triangle to the opposite side. The proposition that it is impossible to have in a plane two equal and distinct triangles ABC and ABC' lettered in the same order with common side AB and lying on the same side of AB, serves as lemma for the usual theorems concerning equal triangles. The theorems on parallels and transversals are proved by means of central symmetry. The ratio of two incommensurables A, B is established by means of a series of inequalities as in the following example:

$$B < A < 2\,B$$
$$1.4\,B < A < 1.5\,B$$
$$1.41\,B < A < 1.42B$$
$$1.414\,B < A < 1.415B$$
$$\text{etc.}$$

In this case $A = \sqrt{2}\,B$, where the $\sqrt{2}$ is defined by the series of inequalities

$$1 < 2$$
$$1.4 < 1.5$$
$$1.41 < 1.42$$
$$1.414 < 1.415$$
$$\text{etc.}$$

Circumference and area of circle are discussed in terms of regular polygons on an intuitive basis; a rigorous treatment based on the concept of a straight line as a continuum of points is given in outline.

APPENDIX B

Questionnaire Based on Major Ideas Gleaned from Synopses in Appendix A

Key to answers received from members of the Committee on Geometry: *y*=yes, *n*=no, ?=doubtful, *b*=blank. Under each answer appears a brief summary of opinions of 101 other teachers.

1. Can the important *facts* of geometry be mastered below the tenth grade, presumably by instruction in informal geometry?	18 y, 2 n, 2 ?, 1 b, 101—majority assent
2. Ought the informal geometry of the junior high school to include *inductions* based	
a) on measurement?	21 y, 1 n, 1 b
b) on constructions with ruler and compasses?	20 y, 1 n, 2 b
c) on cutting and pasting?	20 y, 1 n, 2 b
d) on observation (i.e. contemplation of diagrams and designs)?	20 y, 1 n, 2 b 101—large majority assent to a, b, c, d.
3. Ought this informal geometry to include simple *deductions* based	
a) on *inductions* from observation and study of diagrams?	18 y, 1 n, 2 ?, 2 b
b) on geometric notions *intuitively* held?	18 y, 0 n, 2 ?, 3 b 101—majority assent to a, b.
4. Ought this informal geometry to include	
a) an elementary treatment of loci (without mentioning the technical term)?	17 y, 3 n, 1 ?, 2 b
b) the construction of a triangular prism?	15 y, 4 n, 1 ?, 3 b
c) the construction of a square pyramid?	15 y, 4 n, 1 ?, 3 b
d) the construction of a cylinder?	15 y, 4 n, 1 ?, 3 b
e) the construction of a cone?	15 y, 4 n, 1 ?, 3 b 101—majority favor a, b, c, d, e.

f) the construction of the regular tetrahedron and octahedron?	10 y, 8 n, 1 ?, 4 b
g) the construction of the regular dodecahedron?	6 y, 11 n, 2 ?, 4 b
h) the construction of the regular icosahedron?	6 y, 11 n, 2 ?, 4 b 101—majority oppose f, g, h, though strong minority favor.
5. Ought this informal geometry to include the computation of areas and volumes of	
a) prisms?	20 y, 1 n, 0 ?, 2 b
b) pyramids?	19 y, 1 n, 1 ?, 2 b
c) cylinder?	20 y, 1 n, 0 ?, 2 b
d) cone?	19 y, 1 n, 1 ?, 2 b
e) sphere?	18 y, 1 n, 2 ?, 2 b 101—large majority favor a, b, c, d, e.
6. Ought this informal geometry to include computation of the ratio	
a) of corresponding lengths in similar solids?	11 y, 7 n, 3 ?, 2 b
b) of corresponding areas in similar solids?	11 y, 7 n, 3 ?, 2 b
c) of corresponding volumes in similar solids?	11 y, 7 n, 3 ?, 2 b 101—majority favor a, b, c though considerable minority oppose.
7. Ought this informal geometry to include recognition and acceptance of the fact	
a) that a series of parallels cuts off proportional segments on two transversals (without the technical terminology)?	8 y, 10 n, 3 ?, 2 b 101—majority favor but strong minority oppose.
b) that a plane parallel to the base of a pyramid (or cone) produces two similar pyramids (or cones)?	3 y, 11 n, 6 ?, 3 b 101—about equally divided pro and con.

c) that if three lines are mutually perpendicular, each line is perpendicular to the plane of the other two?	4 y, 11 n, 5 ?, 3 b
d) that if two planes are perpendicular to a third plane, their intersection is perpendicular to the third plane also?	4 y, 11 n, 5 ?, 3 b 101—majority oppose, but strong minority favor c, d.
e) that the difference in longitude and time between two places on the earth can be computed in terms of the arc of the equator included between the meridians of the two places?	8 y, 7 n, 5 ?, 3 b 101—majority favor though many oppose. Note: 101 teachers more favorable than the committee to no. 7.
8. Ought this informal geometry to show the applications of geometry to	
a) the building of wooden dwellings, modern skyscrapers, bridges, railroads, and tunnels?	20 y, 0 n, 1 ?, 2 b
b) the fashioning of garments?	16 y, 1 n, 2 ?, 4 b
c) the designing of decorative motifs?	20 y, 1 n, 0 ?, 2 b 101—large majority favor a, b, c.
9. a) What fractional part of the total junior high school triennium ought to be devoted to informal geometry?	Range 1/10 to 1/3, median 1/5 101—same range, median 1/6.
b) Ought this to be spread over all three years?	11 y, 12 n
c) Or to be confined to two grades: (*Indicate which two*)	4 in 7, 8; 2 in 8, 9
d) Or confined to one grade? (*Indicate which one*)	2 in 7; 2 in 8; 2 in 9 101—bare majority favor b; others prefer most of the geometry in grade 8.
10. a) Ought we to spread the instruction in *demonstrative* geometry over two or more successive years? (This means a more attenuated	5 y, 15 n, 2 ?, 1 b 101—majority oppose.

course taught probably in parallel with other mathematics.)	
b) Ought the main outcomes of *demonstrative* geometry to pertain to logical thinking?	15 y, 3 n, 1 ?, 4 b 101—yes, few dissent.
(Before answering c) read c)—f).	
c) Ought we to continue our present practice of keeping informal (i.e. constructional, mensurational, observational, inductive, intuitive) geometry essentially distinct from demonstrative (i.e. logical, deductive) geometry? (Under this arrangement the facts learned inductively in the informal course appear a second time in the logical presentation of demonstrative geometry.)	8 y, 5 n, 10 b 101—nearly all assent.
d) Or ought our entire course of study in geometry to cover the facts of geometry but once, passing gradually from the informal methods to the formal demonstrative methods? (This limits demonstrative geometry to the logical consideration of single theorems, and to chains of theorems, but does not permit geometry to be viewed as an example—somewhat imperfect—of an abstract logical system.)	1 y, 11 n, 11 b 101—those who answered oppose.
e) Or ought we to compromise between c) and d), introducing simple deductions in the informal stage (see question 3 again) and embellishing the demonstrative stage with a wealth of examples of the applications of geometry?	14 y, 0 n, 2 ?, 7 b 101—majority assent, a few oppose.
f) Or ought we to recognize three stages in our instruction in geom-	7 y, 4 n, 1 ?, 11 b 101—majority assent.

etry, *informal*, *transitional*, and *systematic*, as set forth in the British Report on the Teaching of Geometry in Schools, London, 1917, and in the recommendations of the Committee of the N.E.A. on College Entrance Requirements, 1899?	Interpretation of replies to c, d, e, f: teachers of geometry oppose d, preferring to maintain a distinction between geometric fact and demonstration; they do favor gradual transition from fact to proof, but prefer to get this by adding simple deductions to the informal stage rather than by constructing a separate middle stage. Nevertheless, and somewhat contradictorily, they are not opposed to the establishment of a separate middle stage to serve as transition.
11. Ought demonstrative geometry	
a) to emphasize originals and minimize book theorems?	16 y, 5 favor both, 2 b A few warn against minimizing theorems, suggest treating book theorems as originals. 101—yes
b) to *begin* with the pupil's effort to prove originals or to prove book theorems?	15 orig., 4 theor., 1 ?, 3 b 101—majority blank, others favor theorems.
c) to reveal the logical structure of individual theorems by means of *analyses* preceding the synthetic proofs?	19 y, 2 n, 1 ?, 1 b 101—large majority favor.
12. Ought demonstrative geometry to call attention	
a) to logical chains of theorems?	21 y, 1 n, 1 b 101—favor unanimously.

b) to gaps in Euclid's logic, e.g. his tacit assumptions?	13 y, 4 n, 4 ?, 2 b 101—on the whole yes, but not for poor students.
c) to show pupils the nature of a mathematical system, the need of undefined terms, the arbitrariness of assumptions, the possibility of other arrangements of propositions than that given in their own text?	17 y, 3 n, 3 ? 101—yes, nearly unanimously.
13. Ought demonstrative geometry	
a) to temper logical rigor with considerations anent the learning process and to widen the list of assumptions so as to include theorems the truth of which seems incontestable to the pupil?	19 y, 1 n, 1 ?, 2 b 101—strongly favor assuming obvious truths.
b) e.g. to postulate the congruence theorem concerning two sides and the included angle?	13 y, 8 n, 1 ?, 1 b 101—only small minority oppose.
c) to postulate the congruence theorem concerning a side and the two adjacent angles?	same 101—same.
d) to postulate the congruence theorem concerning three sides?	10 y, 10 n, 2 ?, 1 b 101—majority oppose.
e) to postulate the theorems concerning central angles and their corresponding arcs?	15 y, 6 n, 1 ?, 1 b 101—yes, nearly unanimous.
f) to postulate other theorems in order to avoid proofs by superposition?	10 y, 8 n, 4 ?, 1 b 101—little objection.
14. If superposition is not repugnant to you, should you wish to make free use of translation, rotation, and symmetry in proving theorems?	14 y, 4 n, 2 ?, 3 b 101—large majority favor.
15. How should you regard the proposal to postulate one of the theorems on similar triangles and derive all the propositions on parallels therefrom,	6 y, 10 n, 5 ?, 2 b 101—22y, 48 n, 15 ?, 16 b Committee prefers to

giving the same prominence to similarity as to equality, and assigning parallelism to a lesser role?	postulate propositions concerning corresponding angles and parallel lines; some even would begin geometry with parallels, turning later to congruence.
16. a) Should you wish to make free use of algebra in demonstrative geometry?	21 y, 2 b 101—favor nearly unanimously.
b) Should you wish to limit the use of algebra to situations involving only rational numbers, i.e., to commensurable cases, or should you wish to employ irrational numbers also *in computations?*	16 favor irrationals, 4 oppose, 2 ?, 1 b 101—majority favor irrationals; some would withhold them from poor students.
c) Should you wish to refer to incommensurables in geometry merely to illustrate the distinction between rational and irrational numbers?	12 y, 6 n, 4 ?, 1 b 101—many blanks, many doubts.
d) Or are you content to disregard incommensurables entirely?	2 y, 17 n, 1 ?, 3 b 101—majority against ignoring incommensurables except with weak students.
e) Should you favor a treatment of incommensurables which depended on the pupil's earlier experience with irrational numbers instead of on a "theory of limits"?	12 y, 4 n, 4 ?, 2 b, 1 wants both 101—many blanks, many doubts tending to favor theory of limits.
17. Would you interrupt the logical development of demonstrative geometry with a treatment of numerical trigonometry in connection with similar triangles, or should you prefer that this topic had been covered earlier in intuitive geometry or algebra?	13 would cover trigonometry earlier, 9 would include it in demonstrative geometry, 1 blank; 3 say trigonometry does not interrupt demonstrative geometry. 101—67 would cover trigonometry earlier.

18. a) Should you object to occasional pertinent references to solid geometry—mainly of a mensurational sort—in a course in plane demonstrative geometry?	2 y, 19 n, 2 b 101—nearly all favor some reference to solid geometry.
b) Or do you desire to merge plane and solid geometry even more intimately?	7 y, 9 n, 3 ?, 4 b 101—opinion equally divided.
c) Or do you prefer to keep them *entirely separate?*	3 y, 12 n, 2 ?, 6 b 101—nearly all oppose complete separation.
19. If the course of study in secondary mathematics could include *either* demonstrative geometry *or* analytic (coordinate) geometry, but not both, which should you prefer?	20 dem., 1 b 101—almost unanimously favor demonstrative geometry.
20. Would it be wise to enrich the content of the exercises in geometry by introducing a few simple ideas from "modern" geometry? For example, radical axis, center of similitude, central projection, inversion?	12 y, 9 n, 2 ? 101—majority oppose except for superior pupils.
21. a) A certain experiment seems to show that demonstrative geometry—as now taught—trains the pupils' ability to reason little or no better than certain commercial subjects. Do you think that demonstrative geometry *can* be so taught that it will develop the power to reason logically more readily than other school subjects?	21 y, 1 ?, 1 b 101—large majority assent, a few dissent.
b) If not, what justification is there for continuing to teach demonstrative geometry?	8 blank, 5 "none or little," 2 "to show abstract logical system," 4 "appreciation," 5 "factual content." 101—geometric facts, space concept, cultural value.
c) If you still see educational possi-	Nearly all say "yes,"

bilities in demonstrative geometry, do these depend on a certain "transfer" of the logical training of geometry to situations outside geometry?

many qualifying their replies to indicate that transfer is not automatic, and that it concerns principally methods, ideals, attitudes. 101—similarly.

d) Do teachers of geometry ordinarily teach in such a way as to secure the transfer of those broader attitudes and appreciations which are commonly said to be most easily transferrable?

Almost unanimously, "Except for a few good teachers, a sorrowful No." 101—similarly, but attributing greater success to teachers of experience.

e) If not, how ought they to modify their ordinary methods in order to secure this transfer?

"Bring logical method to the forefront of consciousness; teach for transfer." "Consciously teach the things we want." "Actually do transferring from geometry to other fields." "Apply forms of reasoning to non-mathematical situations." 'Know both applications and appreciations." "Point out parallel between thinking in geometry and in other fields, and practice both." "Bring in illustrations to show the place of logical thinking in life." "Pay more attention to originals and to analysis."

22. a) When we speak of the function concept with regard to geometry do we have in mind the factual or

12 factual, 3 logical, 2 both, 3 ?, 3 b. 101—1/3 factual; 1/3

	the logical aspects of geometry?	logical, both, or doubtful; 1/3 blank.
	b) How would you link the idea of functionality with the logical abstractions of geometry?	"Cannot be done except by stretching the idea of functionality to include the idea 'varying hypothesis varies conclusion'." "Every conclusion is a function of a condition." Two others, "I wouldn't."
23.	a) Ought geometry to be taught in the 10th, 11th, 12th grades in technical curricula?	19 y*, 1 ?, 3 b 101—yes**
	b) Ought the geometry of the 10th, 11th, 12th grades in technical curricula to stress the logical or the factual outcomes?	15 both, 3 logic, 5 b 101—majority say both.
	c) Ought geometry to be taught in the 10th, 11th, 12th grades in industrial curricula?	17 y,*** 1 ?, 5 b 101—yes.
	d) Ought the geometry of the 10th, 11th, 12th grades in industrial curricula to stress the logical or the factual outcomes?	8 both, 3 logic, 7 fact, 5 b 101—small majority favor fact.
	e) Ought geometry to be taught in the 10th, 11th, 12th grades in commercial curricula?	12 y, 4 n, 2 ?, 5 b 101—majority say no.
	f) Ought the geometry of the 10th, 11th, 12th grades in commercial curricula to stress the logical or the factual outcomes?	8 both, 4 logic, 3 fact, 3 ?, 5 b 101—divided opinion.
	g) Ought geometry to be taught in the 10th, 11th, 12th grades in home economics curricula?	11 y, 5 n, 1 ?, 6 b 101—majority say no.
	h) Ought the geometry of the 10th, 11th, 12th grades in home economics curricula to stress the logical or the factual outcomes?	6 both, 4 logic, 4 fact, 9 b 101—if taught, then logic.

* Of these 9 wanted geometry in all three grades.
** Two-fifths of these wanted geometry in all three grades.
*** Of these 5 wanted geometry in all three grades.

Abstracts of Recent Articles on Mathematical Topics in Other Periodicals

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

Algebra

1. Edgett, George L. *The irrational number*. National Mathematics Magazine. 9: 193–96. April 1935.

An elementary exposition of the history and meaning of irrational numbers.

2. Freilich, Julius. *Save our algebra souls*. High Points. Vol. 17. March 1935, pp. 25–27.

A description of a method of teaching algebra by means of a "mastery chart." The writer determined at first what skills were essential for the first term of algebra and then proceeded to make a chart with the names of all the pupils in the class, providing as many columns alongside each name as there were skills to be mastered. Having made the subject matter meaningful, he "began the teaching process, after which came drill, review, automatizing, testing, and finally reteaching only those points diagnosed from the test to be weak, in the order mentioned."

3. Gore, G. D. *A note on the representation and evaluation of powers of i, where $i=\sqrt{-1}$*. School Science and Mathematics. 35: 476–78. May 1935.

The writer takes issue with the method proposed by L. R. Posey in the November 1934 number of School Science and Mathematics. (Reported in this department in January 1935.) He advocates, instead, the use of the coordinate plane and of the unit circle, and points out the mathematical advantages of the graphic procedure.

4. Posey, L. R. *A correction concerning i^n*. School Science and Mathematics. 35: 512. May 1935.

A reply to Professor Richert's article that appeared in the February 1935 issue of the same magazine. "The formulas are not to be memorized as such. They simply state in symbolic language the rules developed."

5. Weissman, H. *A bright class in intermediate algebra*. High Points. Vol. 17, May 1935, pp. 43–44.

Mr. Weissman is well known for his pioneering attempt in giving the brighter pupils an enriched syllabus in geometry without segregating them into special classes. (See his article, "*Grouping in Geometry Classes*" The Mathematics Teacher, February 1929. 22: 93–108.)

In the article under review he describes an enriched syllabus in intermediate algebra that was used for students who were segregated into one class because of their unusual ability and interest in previous courses in mathematics. In addition to the required topics, the syllabus contains a number of supplementary subjects in advanced algebra and in the history of mathematics, a detailed list of which is included. Some of the additional subject matter was taken up by the class as a whole, while the others were presented by students in weekly talks.

Analytic Geometry

1. Crawford, Lawrence. *Determination of the focus and directrix of a parabola whose equation is given with numerical coefficients.* The Mathematical Gazette. 19: 87–89. May 1935.

In the issue of the same magazine for February 1934 the author described a method for determining the foci etc. of a conic in general and showed that the technique could be applied to the parabola. In this article he shows that for the parabola a direct solution can be put down more quickly.

Arithmetic

1. Connelley, Russel L. *Two review tests for May.* The Instructor. 44: 39+. May 1935.

The second of the two tests deals with arithmetic and consists of two parts. Part I contains 10 problems designed to test the pupils' ability to discover which process should be used for the correct solution. The pupil is *not* required to get the answer but merely to indicate with the appropriate abbreviation which one of the four processes he should use.

In part II the pupil is given 15 problems and is asked not only to indicate which of the two processes, multiplication or division, he should use but to supply the correct multiplier or divisor as well.

2. Crocker, Gladys Hosmer. *Number stories.* The Grade Teacher. Vol. 52. April 1935, pp. 26+. May 1935, pp. 28+. June 1935, pp. 25+.

The continuation of a series of stories to teach number facts and concepts in the primary grades.

In the April number, "Johnie learns the value of a quarter" and "Johnie learns what a foot rule is."

In the May issue "Johnie learns how many eggs in one dozen," and "Johnie learns how much is a pound."

In the June issue "Johnie learns what a yard means."

3. Jennings, Rosalie. *Keeping store—a project in arithmetic.* School and Community. 21: 111–12. March 1935.

A description of a project intended to give practice in arithmetical facts and processes and in the inculcation of desirable social traits such as courtesy, honesty and fair dealing.

4. Scott, Lucy. *Teaching areas and volumes in intermediate grades.* Midland Schools. 49: 214–15+. March 1935.

A comparative study of two methods of teaching areas and volumes. The advantages and shortcomings of each are indicated; the questions to be asked and the procedure to be followed are given explicitly and in great detail.

5. Stelson, H. E. *Business arithmetic for the high school.* School Science and Mathematics. 35: 586–97. June 1935.

A discussion of the various meanings that the term business arithmetic has, and an enumeration of the many topics that are usually included, and a list of those that should be included in text books and courses dealing with it.

6. Wilson, Guy M. *Paying for useless arithmetic.* Education. 55: 428–30. March 1935.

A radio address in which the speaker argued for the following program of teaching arithmetic:

a. "Omit all drill from grades 1 and 2, replacing it with meaningful number games and activities.

b. "Omit traditional and useless processes as indicated by adult usage.

c. "Teaching what remains on a

motivated, systematic and meaningful basis.

d. "The child is entitled to a program of success in useful arithmetic."

Calculus

1. *The first encounter with a limit.* The Mathematical Gazette. 19: 109–23. A Letter from F. C. Boon, 131–32. A Letter from N. M. Gibbins, 132–34. May 1935.

The report of a discussion at the annual meeting of The Mathematical Association (England) held on January 8, 1935.

The material is too rich and the examples given too varied to be adequately summarized in the space at our disposal. It is highly recommended to teachers of mathematics in general and to those of advanced algebra and calculus in particular.

2. Picken, D. K. *On differentials.* Mathematical Gazette. 19: 79–86. May 1935.

The author claims that the standard definition of "differentials" is quite unsound and proceeds to introduce a *non-quantitative* conception of the same idea which he developed in his own teaching. He asserts furthermore that this non-quantitative conception of differentials not only gives a useful and even powerful way of expressing limit results from relations in terms of difference-*quantities* but is adequate to all the facts and is as simple as the essential subtlety of such ideas permits.

Geometry

1. Anonymous. *What pupils think about demonstrative geometry.* High Points. Vol. 17. March 1935, pp. 31–37.

A report of the answers that the pupils in a New York City high school gave to a questionnaire on the reasons for their likes and dislikes of geometry.

2. Foran, T. G. and O'Hara, Brother Colombiere. *Sex differences in achievement in high school geometry.* School Review. 43: 357–62. May 1935.

"The test used was the Webb Geometry Test, Form A. All the pupils whose scores are considered were given the Terman Group Test of Mental Ability, Form A, within a week of the Geometry Test. The tests were administered during the school year 1933–34 in the Catholic high schools of an eastern city where a school survey was conducted."

From the scores made by a group of 436 boys and a group of 437 girls with comparable intelligence, the authors draw the following conclusions:

a. "Boys excel girls in achievement in plane geometry in all the phases of the subject included in the Webb Geometry Test with the exception of knowledge of fundamental definitions in which the two groups are probably equal.

b. "The sex differences extend throughout all levels of intelligence.

c. "On the whole relative variability is greater for the scores of the girls than for those of the boys."

A bibliography of 8 items on the same subjects is appended.

3. Funk, J. C. *The case of plane geometry.* California Journal of Secondary Education. 10: 313–16. April 1935.

The writer points out numerous examples in geometry which would arouse the interest of students to a high pitch, and adds that ". . . it would be most unfortunate, even tragic, if a number of our youth who need geometry would not make its acquaintance or should ever be barred from such an important subject."

4. Knapp, Wallace R. *Little "tinkering" in geometry.* High Points. Vol. 17. April 1935, pp. 16–20.

A description of numerous geometric models that can be made out of "Tinker-toy" dowels and spools. Details are supplied both as to the modes of construction as well as to the theorems to be illustrated.

5. Mallison, H. V. *The use of signs in geometry*. The Mathematical Gazette. 19: 124–30. May 1935.

The author advocates the introduction of the ideas of directed lines and angles into elementary geometry and points out the advantages, both logical and pedagogical, that may be derived from such an innovation.

He laments, furthermore, the lack of universal agreement on conventions of signs among writers on three dimensional analytic geometry and differential geometry.

6. Stewart, Jas. W. *If the bisectors of the base angles of a triangle are equal the triangle is isosceles*. The Mathematical Gazette. 19: 144–45. May 1935.

Another proof of the classic problem in plane geometry, and a bibliography of other demonstrations.

7. Zang, Samuel F. *The handling of a bright group in second term geometry*. High Points. Vol. 17. June 1935, pp. 53–54.

This article is another welcome proof that we are at last awakening to the importance of making provision for the bright pupil by giving him an enriched program. The writer describes the method of segregation as well as the scope of work covered, and concludes with a list of benefits derived both by teacher and pupil from such an arrangement.

Trigonometry

1. Grossman, Howard D. *Note on the expansion of Sin($A \pm B$) and Cos ($A \pm B$)*. School Science and Mathematics. 35: 670. June 1935.

An interesting derivation of the above formulas by means of De Moivre's Theorem.

Miscellenaeous

1. Altshiller-Court, Nathan. *Art and mathematics*. Scripta Mathematica. 3: 103–11. April 1935.

This article is a welcome exception to the usual run of vapid and irresponsible comments on the relation between art and mathematics. Though interested primarily in showing what is common to two fields that are seemingly unrelated, the author never forgets to remind the reader that there are fundamental differences as well. The clarity of style and wealth of analogies make this essay one of the best on a very treacherous subject.

2. Bernstein, Selig, and Reiner, Harry. *A mathematics club paper*. High Points. Vol. 17. June 1935, pp. 68.

A brief description of the important role that a paper, published by the mathematics club, can play in the development of the latent powers and talents of the brighter students.

3. Breslich, E. R. *Mathematics for grades seven, eight, and nine*. School Science and Mathematics. 35: 526–36. May 1935.

After conceding the force of many of the arguments against the traditional teaching of mathematics, the writer reports on an experiment in the reconstruction of syllabi that the department of mathematics of the University of Chicago has conducted for a number of years. The courses for the seventh, eighth, and ninth years are given in some detail with special mention of the objectives, procedure, time, etc. The advan-

tages of the proposed curriculum are also touched upon.

4. Charlesworth, H. W. *Mathematics in the integrated curriculum.* School Science and Mathematics. 35: 622–26. June 1935.

The author believes "that the integrated curriculum that finally comes to pass will still allow for some differentiation of courses . . . that the integrated curriculum will allow certain highly specialized fields such as science and mathematics to conduct courses in these fields separate from the integrated courses."

The author further points out the advantages that will accrue from an integrated program; but he also indicates the circumstances that will militate against a complete carrying out of all its details.

5. Coquillette, L. W. *Related shop mathematics.* Industrial Arts and Vocational Education. 24: 75–76. March 1935.

"The real objective of the related mathematics is to give the apprentice the tools for solution of shop problems, but with it must come confidence in himself that he can project his solution into the shop situation and be sure that his result will be accurate both in method and in dimension."

6. Drushel, J. Andrew. *Comments on some recent mathematical text books.* Junior-Senior High School Clearing House. 9: 565. May 1935.

Short but illuminating comments on a few mathematical books that appeared in 1933–1934.

7. Dwyer, Paul S. *The crisis in high school and college mathematics.* School Science and Mathematics. 35: 492–506. May 1935.

The author's remarks center mainly around the following points:

a. The present tendency is to give mathematics a progressively less important place in high school and college.
b. The continued progress of knowledge demands, on the other hand more and more mathematical training.
c. Certain changes in procedure and content should be made.
d. Many of the recommended changes have already been introduced at Antioch College, and a brief report is given of the techniques that have been utilized in putting them into effect there.

8. Greene, Lydia Elaine. *A century of progress in secondary school mathematics, 1834–1934.* Peabody Journal of Education. 12: 220–32. March 1935.

The purpose of the article is "to contrast with respect to content and method of teaching, the mathematics of the secondary school of 1834 with the mathematics of the secondary school of 1934." A good bibliography is included.

9. Howland, W. E. *Mathematics in civil engineering.* School Science and Mathematics. 35: 351–60. April 1935.

A detailed but lively account of the types of mathematical problems that arise in the practice of civil engineering.

10. Jablonower, Joseph. *Mathematical teaching in the next ten years.* Progressive Education. 12: 338–41. May 1935.

The writer believes that in the next ten years teachers of mathematics will do, as they have never done before, the following three things:

a. "They will find out, for themselves, at any rate, why mathematics should be taught.

b. "They will try to shape the curriculum in terms of these reasons.

c. "They will in this endeavor find it necessary to call for a better preparation on their own part for this work."

Mr. Jablonower also lists what he considers "the three most important realizable outcomes of mathematical teaching, outcomes that are at the same time more nearly true to the nature of mathematics itself.

a. "The notion of mathematics as a means of describing time and space aspects of the phenomenal world.

b. "The notion that the concepts of mathematics have a history and are understood the more clearly in terms of their historical development,

c. "The notion that mathematical work is often the occasion for making explicit the method of reasoning."

He also gives the general outline of a curriculum of the future that will realize the objectives enumerated above, and indicates some tendencies in the present that seem to point the way.

11. Keyser, Cassius J. *Mathematics and the dance of life.* Scripta Mathematica. 3: 120–31. April 1935.

An address delivered at the banquet of the National Council of Teachers of Mathematics, on February 23, 1935, at Atlantic City. "All the arts, including that of the mathematician, are members of one family. All are children of one impulse—the impulse to make. All are partners in the Dance of Life."

12. Lorenz, Aloysius. *Mathematics as an aid to mental training.* The Catholic Educational Review. 33: 282–87. May 1935.

This article might have been written in the days when the faculty theory of the human mind held sway and the problems of transfer of training and specificity of teaching did not agitate the minds of our educators and psycologists. The writer believes that "the study of mathematics aids also in the development of the power of application" and that it "exerts a very great influence on the character of the individual . . . and it trains the will to be more persevering in its efforts to grapple with difficult life problems."

It is, of course, not incumbent upon any one to accept the conclusions of contemporary psychologists upon the impossibility of training vague, general attitudes and habits, such as "character" and "will," unless they are analyzed first into their constituent elements and each of these trained and inculcated specifically and directly. It is, however, customary that when a writer harks back to a theory that is generally, almost universally, discredited, he should at least make some reference to the current theories and give his reasons for rejecting them.

13. Minarik, R. G. *Are we teaching engineering mathematics?* The Journal of Engineering Education. 25: 499–509. March 1935.

A formulation of the problems that beset a conscientious teacher of engineering mathematics, both as to subject matter and to procedure of presentation. What the author calls the "engineer's method of analyzing problems" is applicable as well, with slight modification, to other human beings in mathematical situations and it is, therefore, reproduced below.

a. "Recognize a problem
b. "Analyze it in terms of fundamental principles or laws
c. "Express the analysis mathemetically.
d. "Manipulate the mathematical expressions until some definite objective (the answer) has been reached.
e. "Appreciate the significance and limitations of the mathematical solution.
f. "Evaluate the solution numerically for the specific problem at hand."

14. Moffitt, Roy M. *The mathematics and science behind air conditioning.* School Science and Mathematics. 35: 416–23. April 1935.

An exciting but clearly analyzed account of the role that mathematics and science are playing in the solution of problems that arise in the air conditioning of buildings. "My object is to show you that the new industry fairly reeks with science and is saturated with mathematics."

15. Parsons, G. L. *The work of a Junior Mathematical Association.* The Mathematical Gazette. 19: 65–72. May 1935.

The objects of the Association are threefold:

a. "To increase interest in mathematics [among the students] by lectures on matters not necessarily included in the normal syllabus, and in general to stress the unity and variety of the subject.
b. "To promote and stimulate interchange of ideas between the schools, both among masters and boys.
c. "To endeavor to promote the research spirit, e.g. by entrusting some of the papers to be read by competent boys."

Meeting are held five times a year and the lecturers are equally divided between outside and boy lecturers. Essay competitions and "problem drives" are also conducted to interest those students who need stimulants other than ungovernable curiosity.

16. Reeve, W. D. *The teaching of mathematics in the schools.* Scripta Mathematica. 3: 138–42. April 1935.

In discussing the vital problems of the training of our future teachers of mathematics the writer makes the following points:

a. The greatest need of teachers of mathematics today is a more adequate knowledge of the subject which they are expected to teach—, more scholarly teachers.
b. Mere knowledge of subject matter does not guarantee that the possessor thereof will be a successful teacher.
c. ". . . much of the trouble that we are now in is due to the lack of cooperation between those who should be most vitally concerned with the training of the mathematics teachers of the future. The college and university professors of mathematics have not been enough concerned about the education of teachers in their fields, and as a result what should have been the duty of these professors has been usurped by the professors of education. As a result a sort of bad feeling has grown up. . . . The college and university professors of mathematics are losing some of the best students they can ever hope to secure, and the pupils in the schools are losing some of the

> best teachers they have ever had, because those of us who believe in the value of mathematics in the schools are not doing our duty."

Interesting excerpts are given from a letter by Prof. Einstein and from an article by Prof. E. R. Hedrick on "Desirable cooperation between educationist and mathematician" that appeared in School and Society on December 17, 1932, 36: 769–77.

17. Silberstein, Nathan. *How to study mathematics*. High Points. Vol. 17. April 1935, pp. 54–58.

The writer points out that in addition to his usual duties "the good teacher must always be able to give his pupils a technique whereby they can successfully assimilate what has been taught." He then proceeds to make some excellent suggestions on the best ways of advising pupils to study algebra and geometry.

Unfortunately the advice given on the solution of problems suffers from the implicit assumption that most problems involve but one unknown and ask but one question. Even a cursory examination of algebraic problems will convince the skeptic that the contrary is the case.

18. Stone, Charles A. *The workbook in mathematics*. School Science and Mathematics. 35: 382–87. April 1935.

An eloquent but unbiased defense of the value of the workbook to the students and teachers of mathematics. The writer substantiates his claims by reporting the results and conclusions of a thesis, undertaken at his suggestion, by J. W. Ramsey on "A study of the value of the workbook in teaching Beginning Algebra."

19. Thomas, Carl R. *What value mathematics?* Nebraska Educational Journal. 15: 96. March 1935.

An appeal to teachers of mathematics "to give time and thought to the value of the subject or subjects they teach, and you will be repaid in increasing meaning of the subject to you, increased interest and success on the part of the pupil—and it may even be possible for you to make a contribution to the advancement of mathematics itself."

20. Seidlin, Joseph, and Webber, W. Paul. *Purpose in teaching mathematics*. National Mathematics Magazine. 9: 202–05. April 1935.

Interesting remarks on various phases of teaching mathematics.

21. Wrightstone, J. Wayne. *Comparison of varied curricular practices in mathematics*. School Science and Mathematics. 35: 377–81. April 1935.

A report of an experiment to determine the relative efficacy of two methods of teaching mathematics: the standard compartmentalized method vs the progressive method based on an activity program.

OFFICIAL FINAL BALLOT

The National Council of Teachers of Mathematics
St. Louis Meeting, January 1, 1936

For President, 1936-38 (Vote for one)

☐ Hildebrandt, Martha Maywood, Ill.	☐ Schlauch, W. S. New York, N.Y.

For Second Vice-President, 1936-38 (Vote for one)

☐ Johnson, J. T. Chicago, Ill.	☐ Kelly, Mary Wichita, Kan.

For Members of the Board of Directors, 1936-39 (Vote for three)

☐ Breslich, E. R. Chicago, Ill.	☐ Kee, Olive A. Boston, Mass.
☐ Charlesworth, H. W. Denver, Colo.	☐ Mallory, Virgil S. Montclair, N.J.
☐ Haertter, Leonard D. St. Louis, Mo.	☐ Shriner, Walter O. Terre Haute, Ind.

Members will mark this ballot immediately and mail same to Edwin W. Schreiber, Secretary, 425 E. Calhoun St., Macomb, Illinois. Ballots sent by mail must reach Macomb by Friday, Dec. 28. The polls will close at 8:00 P.M., Tuesday, Dec. 31. You must use this official printed ballot or send to the Secretary for a ballot.